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Generalized Formula of the Area of N-Point Star (Regular)

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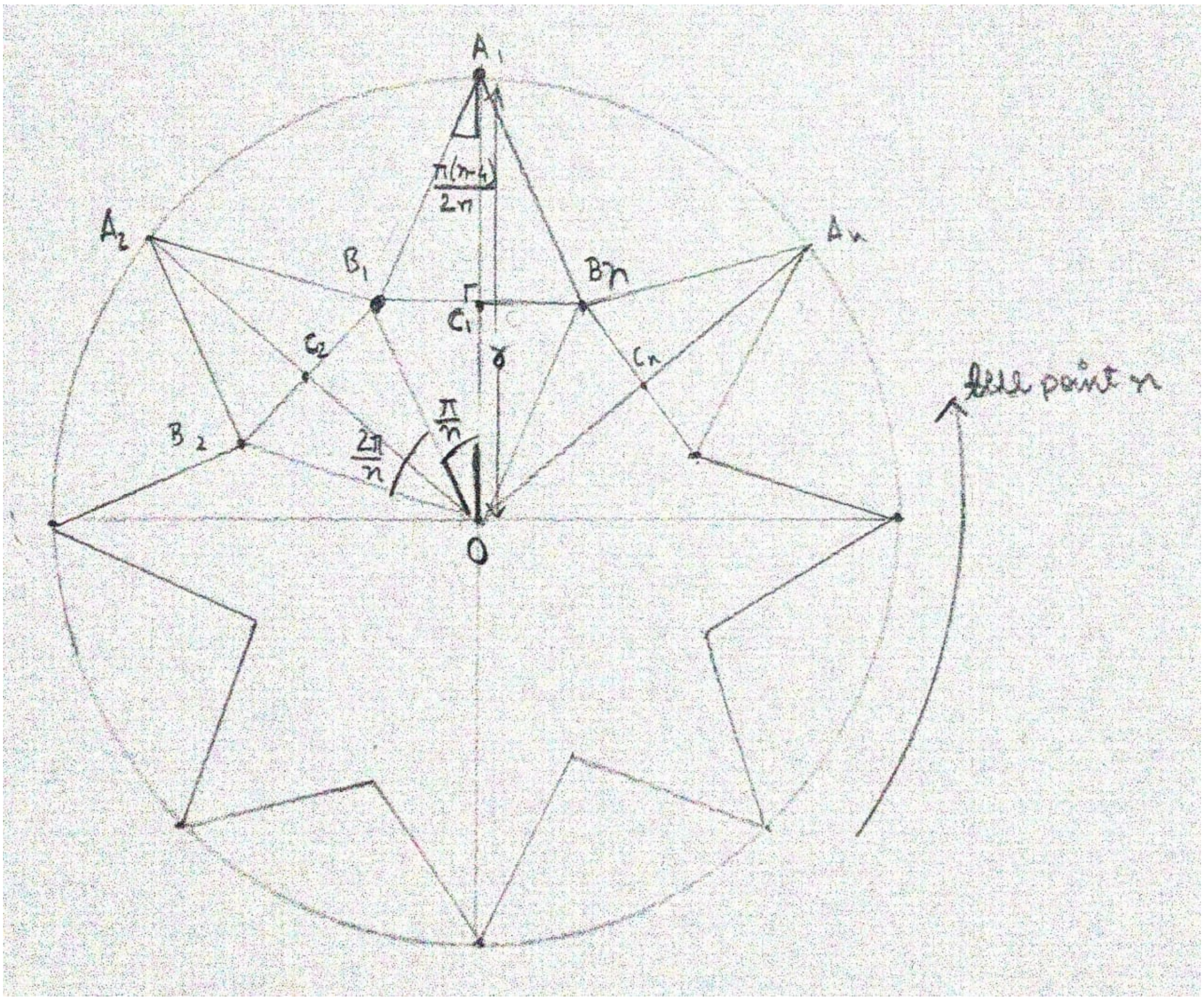
Statement

The area of a regular n-point star is equal to

$nr^2(\tan\frac{\pi(n-4)}{2n}\tan\frac{\pi}{n})/(\tan\frac{\pi(n-4)}{2n}+\tan\frac{\pi}{n})$ where n is the number of points of the star and r is the radius of the circle within which the star lines and whose all points are touching the circle.



Proof

Let,

We put a n-point star inside a circle with centre O and of radius r.

As it is a regular n-point star.

Let,All it's sides i.e A1B1=A2B1=A2B2=……….=AnBn=A1Bn

In ΔA1OB1 and ΔA1OBn

A1O = A1O (=r Given)

A1B1=A1Bn (sides)

OB1=OBn (given)

So, ΔA1OB1 ≅ ΔA1OBn (By SSS congruencecriteria)

Similarly it can be shown that,

ΔA1OB1 ≅ ΔA2OB1 ≅ΔA2OB2 ≅………………≅ ΔA1OBn

So,

∠A1OB1 =∠A2OB1 =∠A2OB2 =…………..=∠A1OBn

(ByCPCT) -------------------------- (1)

And also ∠B1A1O=∠B1A2O=∠B2A2O=………∠BnA1O

(By CPCT) -------------------------- (2)

Now, Sum of all angles around the centre O= 2π

Or ∠A1OB1+∠A2OB1+∠A2OB2+…..+∠A1OBn=2π

Or 2n*∠A1OB1= 2π (From equation (1))

Or **∠A1OB1= 2π/2n= π/n**

Sum of all angles at the points of the star = (n-4) π

Or ∠B1A1O+∠B1A2O+∠B2A2O…+∠BnA1O=(n-4)π

Or 2n*∠B1A1O=(n-4)π (From equation (2))

Or **∠B1A1O= (n-4) π/2n**

In ΔB1A1Bn

B1A1=BnA1

Therefore, ΔB1A1Bn is an isosceles Δ and perpendicular from .A1 to base B1Bn bisects B1Bn.

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So, B1C1=BNC1= x (Suppose)

And B1Bn=B1C1+BnC1=x+x=2x

Also, suppose A1C1=a

And C1O=b

So in right angled ΔA1B1C1,

$\tan\frac{\pi(n-4)}{2n}$= B1C1/A1C1=x/a

Or **a=x/**$\boldsymbol{\tan\frac{\pi(n-4)}{2n}}$ --------------------------(3)

And in right angled ΔOB1C1,

$\tan\frac{\pi}{n}$= B1C1/C1O=x/b

Or **b=x/** $\boldsymbol{\tan\frac{\pi}{n}}$ --------------------------(4)

We know that A1O=A1C1+C1O=a+b=r --------------------------(5)

Putting value of a and b from equations (3) and (4)

We get,

a+b=r

or x/$\tan\frac{\pi(n-4)}{2n}$+x/ $\tan\frac{\pi}{n}$ =r

or x $\left(\tan\frac{\pi(n-4)}{2n}+\tan\frac{\pi}{n}\right)/\left(\tan\frac{\pi(n-4)}{2n}\tan\frac{\pi}{n}\right)$=r

or x=$r\left(\tan\frac{\pi(n-4)}{2n}\tan\frac{\pi}{n}\right)/\left(\tan\frac{\pi(n-4)}{2n}+\tan\frac{\pi}{n}\right)$ -------------------------(6)

Area of the n-point star(Regular)

=Area of (ΔA1OB1+ΔA2OB1+ΔA2OB2+………+ΔA1OBn)

=2n*Area of ΔA1OB1 (As it has been shown that all these Δs are congruent or they have equal area)

=2n*1/2*A1O*B1C1

=n*(A1C1+C1O)*B1C1

=n*(a+b)*x

=n*r*x (putting value of (a+b)=r from equation(5))

=n*r*r($\tan\frac{\pi(n-4)}{2n}\tan\frac{\pi}{n}$) / ($\tan\frac{\pi(n-4)}{2n} + \tan\frac{\pi}{n}$) (Putting value of x from equation (6))

Therefore

Area of a regular n-point star

$$= \boldsymbol{nr^2}\left(\tan\frac{\pi(n-4)}{2n}\tan\frac{\pi}{n}\right) / \left(\tan\frac{\pi(n-4)}{2n} + \tan\frac{\pi}{n}\right)$$

