Young Scientist- Tomorrow's Science Begins Today
Vol. 4, Issue 1 – 2020
ISSN: 2581-4737

In an A.P (of even no. of terms) sum of all product of term taken equidistant from end and beginning is given by $n\left[a^2 + \frac{\{2n-1\}(3a+(n-1)d\}d}{3}\right]$

Kautsabh Tripathi[1]

[1]*Class-X, Semstar Global School, Prayagraj, India.*

Introduction

This formula has been developed to calculate the sum of all product of term taken equidistant from end and beginning of an A.P(of even no. of terms). From this formula we can derive some properties of integers and natural no.

Statement

In an A.P (of even no. of terms) sum of all product of term taken equidistant from end and beginning is given by $n\left[a^2 + \frac{\{2n-1\}(3a+(n-1)d\}d}{3}\right]$

Where n = half the no. terms

a = first term of A.P

d = common difference.

Proof:

Let the A.P be a,a+d,a+2d,- - - - - - - - - - -,a+(2n-1)d.

Clearly it has 2n terms with first term a and common difference of d.

Sum of all product of term taken equidistant from end and beginning = a{a+(2n-1)d}+{a+d}{a+(2n-2)d}+{a+2d}{a+(2n-3)d}+- - - - - - - - - - - - - - - +{a+(n-2)d}[a+{2n-(n-1)d}]+{a+(n-1)d}[a+{2n-n}d]

$\rightarrow a^2$ +ad(2n-1)+ a^2+ad(2n-2)+ad+d^2(2n-2)+ a^2+ad(2n-3)+2ad+2d^2(2n-3)+ - - - - - - - - - - - +a^2+ad{2n-(n-1)}+ad(n-2)+ d^2{n-2}{2n-(n-1)}+ a^2+ad(2n-n)+ad(n-1)+ d^2(2n-n)(n-1)

$\rightarrow na^2$+ad(2n-1+2n-2+2n-3+- - - - - +2n-n)+ad{1+2+3+- - - - - - +(n-1)}+ d^2{[2n+4n+6n - - - - - - $(n-1)^{th}$term}-{2+6+12+20- - - - - -$(n-1)^{th}$term}] 1

Let S = 2+6+12+20- - - - - -$(n-1)^{th}$term 2

S = 2+6+12+20- - - - - -$(n-1)^{th}$term 3

On subtracting equation 3 from 2 we get,

$0 = 2+(4+6+8$ - - - - - - - - - - - $(n-2)^{th}$ term)- $(n-1)^{th}$term

$0 = 2+[\{\frac{(n-2)}{2}\{8+(n\text{-}3)2\}\}]\text{-}\ (n-1)^{th}$term

$T_{(n-1)} = 2+[(n\text{-}2)(n+1)]$

$T_{(n-1)} = 2+n^2\text{-}2\text{-}2n+n$

$T_{(n-1)} = n^2\text{-}n$

So, $(n-1)^{th}$term of series 0+2+6+12+20- - - - - $(n-1)^{th}$term is n^2-n.

Clearly on putting n = 1,2,3 - - - - - - -n we will get each term of series.

$T_{(1-1)} = 1^2\text{-}1$

$T_{(2-1)} = 2^2\text{-}2$

$T_{(3-1)} = 3^2\text{-}3$

||

||

$+T_{(n-1)} = n^2\text{-}n$

$(\frac{n(n+1)(2n+1)}{6}\text{-}\frac{n(n+1)}{2})$

Hence, 2+6+12+20- - - - - $(n-1)^{th}$term = $(\frac{n(n+1)(2n+1)}{6}\text{-}\frac{n(n+1)}{2})$.

Putting $(\frac{n(n+1)(2n+1)}{6}\text{-}\frac{n(n+1)}{2})$ in place of 2+6+12+20- - - - - $(n-1)^{th}$term in equation 1.

$\rightarrow na^2+ad[2n^2\text{-}\ \frac{n(n+1)}{2}]+ad\{\ \frac{n(n-1)}{2}\}+\ d^2[\{\frac{n-1}{2}(4n+(n\text{-}2)2n)\text{-}(\frac{n(n+1)(2n+1)}{6}\text{-}\frac{n(n+1)}{2})$

$\rightarrow na^2+ad[2n^2\text{-}n]+\ d^2[\{n^2(n-1)\text{-}(\frac{n(n+1)(2n+1)}{6}\text{-}\frac{n(n+1)}{2})\}]$

$\rightarrow na^2+adn[2n-1]+\ d^2[\{n^2(n-1)\text{-}(\frac{n(n+1)(n-1)}{3})\}]$

$\rightarrow na^2+adn[2n-1]+\ d^2n[(n-1)(\frac{(2n-1)}{3})]$

$\rightarrow na^2+dn(2n-1)(\frac{3a+(n-1)d}{3})$

$\rightarrow n\left[a^2 + \frac{\{2n-1\}(3a+(n-1)d\}d}{3}\right]$

Corollary 1

Sum of all product of term taken equidistant from end and beginning of first 2n natural no. is given by $\frac{n}{3}(2n+1)(n+1)$.

Proof

Clearly on putting a = 1 and d = 1 we will get desired result.

So,

$\rightarrow n\left[1^2 + \frac{\{2n-1\}(3+(n-1)1\}1}{3}\right]$

$\rightarrow \frac{n}{3}(2n^2+3n+1)$

$\rightarrow \frac{n}{3}(2n+1)(n+1)$

Note: Sum of all product of term taken equidistant from end and beginning of first 2n negative integers is also given by $\frac{n}{3}(2n+1)(n+1)$ as the product of two negative no. is always positive no. .

Corollary 2

Sum of all product of term taken equidistant from end and beginning of first 2n odd no. is given by $\frac{n}{3}(1+8n^2)$.

Proof

Clearly on putting a = 1 and d = 2 we will get desired result.

So,

$\rightarrow n\left[1^2 + \frac{\{2n-1\}(3+(n-1)2\}2}{3}\right]$

$\rightarrow \frac{n}{3}(8n^2+1)$

Note: Sum of all product of term taken equidistant from end and beginning of first 2n negative odd integers is also given by $\frac{n}{3}(1+8n^2)$ as the product of two negative no. is always positive no.

Corollary 3

Sum of all product of term taken equidistant from end and beginning of first 2n even no. is 4 times the sum of all product of term taken equidistant from end and beginning of first 2n natural no. .

Proof

Clearly to prove this we first need to find the sum of all product of term taken equidistant from end and beginning of first 2n even no. and then divide it by sum of all product of term taken equidistant from end and beginning of first 2n natural no. .

On putting a = 2 and d = 2 we will get sum of all product of term taken equidistant from end and beginning of first 2n even no. .

So,

$\rightarrow n\left[2^2 + \frac{\{2n-1\}(6+(n-1)2\}2}{3}\right]$

$\rightarrow \frac{n}{3}\{12+4(2n-1)(2+n)\}$

$\rightarrow \frac{4n}{3}(2n^2+3n+1)$

$\rightarrow \frac{4n}{3}(2n+1)(n+1)$

Sum of all product of term taken equidistant from end and beginning of first 2n natural $= \frac{n}{3}(2n+1)(n+1)$.

$$\frac{\text{Sum of all product of term taken equidistant from end and beginning of first 2n even no.}}{\text{Sum of all product of term taken equidistant from end and beginning of first 2n natural no.}} = \frac{\frac{4n}{3}(2n+1)(n+1)}{\frac{n}{3}(2n+1)(n+1)} = 4$$

Hence, sum of all product of term taken equidistant from end and beginning of first 2n even no. is 4 times the sum of all product of term taken equidistant from end and beginning of first 2n natural no. .

Note: This result is also true for integers.

Corollary 4

Sum of all product of term taken equidistant from end and beginning of first 2n even no. is always greater than sum of all product of term taken equidistant from end and beginning of first 2n odd no.

Proof

To prove this we have to subtract sum of all product of term taken equidistant from end and beginning of first 2n odd no. from sum of all product of term taken equidistant from end and beginning of first 2n even no. .

So,

$\rightarrow \frac{4n}{3}(2n+1)(n+1)- \frac{n}{3}(1+8n^2)$

$\rightarrow$n(4n-1), which is a positive no. as n is always positive.

Hence, sum of all product of term taken equidistant from end and beginning of first 2n even no. is always greater than sum of all product of term taken equidistant from end and beginning of first 2n odd no. .

Note: This result is also true for integers.

