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For any two Natural Numbers which Differ by Two, The Sum of Squares of Number When Divided by their Sum, It Always Gives 2 as Remainder

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Introduction

A theorem has been developed for division and multiplication of two natural numbers with the aim to easy the process.

Statement

For any two natural numbers which differ by two, the sum of squares of number when divided by their sum, it always gives 2 as remainder.

Proof

Let the numbers be- 'a' & 'a+2' [a€ I^+]

Let 'r' be the remainder when $a^2 + (a+2)^2$ is divided by a +(a+2)

$a^2 + (a+2)^2 \equiv r (\text{mod } a+a+2)$ [r€ I+, $0 \le r < \{a+(a+2)\}$]

Now, $a^2 + (a+2)^2 = [a + (a+2)]^2 - 2a(a+2)$

$= 4 + 2(a^2 + 2a) + 2 - 2$

$=2 + 2(a^2 + 2a + 1)$

$=2 + 2(a+1)^2$

$=2 + (2a+2)(a+1)$

⇨ $(2a+2)(a+1) + 2 \equiv r (\text{mod} 2a+2)$

⇨ $2a+2 \mid (2a+2)(a+1) + 2 - r$

Clearly, $2a+2 \mid (2a+2)(a+1)$

⇨ $2a+2 \mid 2-r$

⇨ $2a+2 \mid r-2$

$r \ge 0$ => $(r-2) \ge -2$, $r < 2a+2$ => $(r-2) < 2a+2$

⇨ $-2 \le (r-2) < 2a+2$

Case-1 [$-2\leq (r-2)\leq 0$]

$2(a+1) \mid r-2 \Rightarrow 2 \mid r-2$

⇨ $r-2= -2, 0$

If $(r-2)= -2$

⇨ $2(a+1) \mid -2 \Rightarrow a+1 \mid -1$

⇨ $a+1= \pm 1$ a contradiction $\Rightarrow r-2\neq -2$

Since, every number divides 0,

Thus, $r-2= 0 \Rightarrow r=2$

Case-2[$0< (r-2)< 2(a+1)$]

Since2(a+1)is a factor positive value of (r-2)

⇨ $(r-2)> 2(a+1)$ a contradiction

Thus, no solution.

Therefore, ultimately r=2

Thus, remainder = 2

Corollary

For any two number which differ by two, the sum of squares of the numbers when divided by their sum, it always leaves 2 as remainder and their average as quotient.

PROOF

Let the numbers be- 'a' & 'a+2' [$a\in I^+$]

Dividend = $a^2 + (a+2)^2$, Divisor = $a + (a+2)$, Quotient = Q, Remainder = r [$r,Q\in I^+$]

By division algorithm, we get-

$a^2 + (a+2)^2 = [a+(a+2)]Q + r$

By the above theorem, r=2

$[(a+2)-a]^2 + 2a(a+2) = 2(a+1)Q + 2$

⇨ $2^2 + 2(a^2+2a) - 2 = 2(a+1)Q$

⇨ $4 + 2(a^2+2a) – 2 = 2(a+1)Q$

⇨ $2 +2(a^2+2a) = 2(a+1)Q$

⇨ $2(a^2+2a+1) = 2(a+1)Q$

⇨ $2(a+1)^2 = 2(a+1)Q$

⇨ $Q = a+1 = a+(a+2)/2$ = Average of the numbers

Thus, quotient = average of the numbers

