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ON SOFT GENERALIZED βb-CLOSED SETS IN SOFT TOPOLOGICAL SPACES

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ABSTRACT

In this paper, we introduce a new class of soft generalized closed sets in soft topological spaces called soft $g\beta b$-closed sets is introduced and its soft topological properties is studied and investigated. Moreover, we discussed the relationship among, gb-closed, βg-closed, sw-closed, sg-closed, $s^* g$-closed, wg-closed, gp-closed, g-closed, rwg-closed, gpr-closed, rg-closed, gs-closed and nowhere dense sets. Finally, we defined and discussed the properties of soft $g\beta b$-open sets and $g\beta b$-neighborhood.

KEYWORDS: Soft $g\beta b$-Closed, Soft β-Open Sets, Soft B-Open Sets, Soft B-Interior, B-Closure.

1. INTRODUCTION

The concept of soft set theory has been introduced in 1999 by Molodtsov [1] this set designed to solve the sophisticated problems in economic, engineering, environment, etc. It has been applied to several branches of mathematics such as operation research, game theory and among others.

The soft set theory and it's applications increase after time to several researchers, especially in the recent years. This is because of the general nature of parameterizations expressed by a soft set. Therefore due to these facts, several special sets have been introduced in the soft set theory and their properties have been studied, within the soft topological space. The notion of topological spaces for soft sets was formulated by Shabir and Naz [2], which is defined over an initial universe with fixed set of parameters. Levine [3] introduced generalized closed sets in general topology. Kannan [4] introduced soft generalized closed and open sets in soft topological spaces which are defend over an initial universe with a fixed set of parameters. He studied their some properties. After then Saziye et al. [5] studied behavior relative to soft subspaces of soft generalized closed sets and continued investigating the properties of soft generalized closed and open sets. Nazmul and Samanta [6] introduced neighborhood properties of soft topological spaces. Hussain and Ahmad [7] introduced soft topological spaces and the notions of soft open sets, soft closed sets, soft closure, soft interior points, soft neighborhood of a point and soft separation axioms.

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Soft β-open sets and its properties were introduced and studied by M.E. Abd El-Monsef [8]. Akdag and Ozkan [9] introduced soft b-open sets and soft b- continuous functions. Characterization of b-open soft set in soft topological spaces was introduced and studied by El-Sheikh and El-latif [10]. Let (F, E) be a soft set over X, the soft closure of (F, E) and the soft interior of (F, E) will be denoted by $cl(F, E)$ and $int(F, E)$ respectively, the union of all soft b-open sets over X contained in (F, E) is called soft b-interior of (F, E) and it is denoted by $bint(F, E)$, the intersection of all soft b-closed sets over X contain (F, E) is called soft b-closure of (F, E) and it is denoted by $bcl(F, E)$. In this paper, we introduce a new type of generalized closed set in soft topological spaces. Further, we investigate some soft topological properties of this set.

2. PRELIMINARIES

In this section, we present the basic definitions and results of soft set theory which will be needed in the sequel.

Definition 2.1 [1]

Let X be an initial universe and E be a set of parameters. Let $P(X)$ denote the power set of X and A be a non-empty subset of E. A pair (F, A)denoted by F_A is called a soft set over X, where F is a mapping given by $F : A \rightarrow P(X)$. In other words, a soft set over X is a parameterized family of subsets of the universe X. For a particular $e \in A, F(e)$ may be considered the set of e-approximate elements of the soft set (F, A) and if $e \notin A$, then $F(e) = \Phi$ i.e $F_A = \{F(e) : e \in A \subseteq E, F : A \rightarrow P(X)\}$. The family of all these soft sets over X denoted by $SS(X)_A$.

Definition 2.2 [11]

Let $F_A, G_B \in SS(X)_E$. Then F_A is soft subset of G_B, denoted by $F_A \widetilde{\subseteq} G_B$, if

1. $A \subseteq B$, and

2. $F(e) \subseteq G(e), \forall e \in A$.

In this case, F_A is said to be a soft subset of G_B and G_B is said to be a soft superset of F_A, $F_A \widetilde{\subseteq} G_B$.

Definition 2.3 [11]

Two soft sets F_A and G_B over a common universe set X are said to be soft equal if F_A is a soft subset of G_B and G_B is a soft subset of F_A.

Definition 2.4 [12]

The complement of a soft set (F, A), denoted by $(F, A)^c$, is defined by $(F, A)^c = (F^c, A)$, $F^c : A \rightarrow P(X)$ is a mapping given by $F^c(e) : X - F(e), \forall e \in A$ and F^c is called the soft complement function of F. Clearly $(F^c)^c$ is the same as F and $((F, A)^c)^c = (F, A)$.

Definition 2.5 [13]

The difference of two soft sets (F, E) and (G, E) over the common universe X, denoted by $(F, E) - (G, E)$ is the soft set (H, E) where for all $e \in E, H(e) = F(e) - G(e)$.

Definition 2.6 [13]

Let (F, E) be a soft set over X and $x \in X$. We say that $x \in (F, E)$ read as x belongs to the soft set (F, E) whenever $x \in F(e)$, for all $e \in E$.

Definition 2.7 [13]

Let $x \in X$. A soft set (x, E) over X, where $x_E(e) = \{x\}, \forall e \in E$, is called the singleton soft point and denoted by x_E.

Definition 2.8 [11]

A soft set (F, A) over X is said to be NULL soft set denoted by $\Phi \; or \; \Phi_A$ if for all $e \in A, F(e) = \Phi(null \; set)$.

Definition 2.9 [11]

A soft set (F, A) over X is said to be an absolute soft set denoted by $\widetilde{A} \; or \; X_A$ if for all $e \in$

$A, F(e)$ = X. Clearly we have $X_A^C = \Phi_A$ and Φ_A^C = X_A.

Definition 2.10 [11]

The union of two soft sets (F, A) and (G, B) over the common universe X is the soft set (H, C), where $C = A \cup B$ and for all $e \in C$, $H(e) = \begin{cases} F(e), e \in A - B, \\ G(e), e \in B - A, \\ F(e) \cup G(e), e \in A \cap B \end{cases}$

Definition 2.11 [11]

The intersection of two soft sets (F, A) and (G, B) over the common universe X is the soft set(H, C), where $C = A \cap B$ and for all $e \in C, H(e)$ = F(e) ∩ G(e). Note that, in order to efficiently discuss, we consider only soft sets, (F, E) over a universe X in which all the parameter set E are same. We denote the family of these soft sets by $SS(X)_E$.

Definition 2.12 [10]

Let I be an arbitrary indexed set and $L = \{(F_i, E);\ i \in I\}$ be a subfamily of $SS(X)_E$.

1. The union of L is the soft set(H, E), where $H(e) = \cup_{i \in I} F_i(e)$ for each $e \in E$. We write $\widetilde{\cup}_{i \in I} (F_i, E) = (H, E)$.

2. The intersection of L is the soft set(M, E), where $M(e) = \cap_{i \in I} F_i(e)$ for each $e \in E$. We write $\widetilde{\cap}_{i \in I} (F_i, E) = (M, E)$.

Definition 2.13 [13]

Let τ be a collection of soft sets over a universe X with a fixed set of parameters E, then $\tau \subseteq SS(X)_E$ is called a soft topology on X if

1. $X, \Phi \in \tau$, where $\Phi(e) = \Phi$ and $X(e) = X, \forall e \in E$,
2. The union of any number of soft sets in τ belongs to τ,
3. The intersection of any two soft sets in τ belongs to τ.

(X, τ, E) is called a soft topological space over X.

Definition 2.14 [6]

Let (X, τ, E) be a soft topological space and $(F, E) \in SS(X)_E$. Define $\tau_{(F,E)} = \{(G, E) \widetilde{\cap} (F, E) : (G, E) \in \tau\}$ which is a soft topology on (F, E). The soft topology is called soft relative topology of τ on (F, E), and $\left((F, E), \tau_{(F,E)}\right)$ is called soft subspace of (X, τ, E).

Definition 2.15 [13]

A soft set $(F, E) \in SS(X)_E$ is called a soft point in X_E if there exist $x \in X$ and $e \in E, F(e) = \{x\}$ and $F(e^c) = \Phi$ for each $e^c \in E - \{e\}$, and the soft point (F, A) is denoted by x_e is said to belong to the soft set (G, E), $x_e \widetilde{\in} (G, A)$, if for the element $e \in A, F(e) \subseteq G(e)$.

Definition 2.16 [6]

A soft set (G, E) in a soft topological space (X, τ, E) is called a soft neighborhood of a soft point $F(e)$ if there exists a soft open set (H, E) such that $F(e) \in (H, E) \widetilde{\subseteq} (G, E)$.

A soft set (G, E) in a soft topological space (X, τ, E) is called a soft neighborhood of a soft set (F, E) if there exists a soft open set (H, E) such that $(F, E) \widetilde{\subseteq} (H, E) \widetilde{\subseteq} (G, E)$. The neighborhood system of a soft point $F(e)$ denoted by $N_\tau(F(e))$, is the family of all its neighborhood.

Definition 2.17 [14]

Let (X, τ, E) be a soft topological space and $(F, E) \in SS(X)_E$, (F, E) is said to be b-open soft set if $(F, E) \widetilde{\subseteq} int(cl(F, E)) \widetilde{\cup}\, cl(int(F, E))$ and it's complement is said to be b-closed soft. The set of all b-open soft sets are denoted by $BOS(X, \tau, E)$, or$BOS(X)$ and the set of all b-closed soft sets are denoted by $BCS(X, \tau, E)$, or $BCS(X)$.

Definition 2.18 [15]

Let (X, τ, E) be a soft topological space and $(F, E) \in SS(X)_E$, (F, E) is said to be

1. Soft Preopen set if $(F, E) \widetilde{\subseteq} int(cl(F, E))$.
2. Soft semi open set if $(F, E) \widetilde{\subseteq} cl(int(F, E))$.
3. Soft β-closed set if $int\left(cl(int(F, E))\right) \widetilde{\subseteq} (F, E)$.

Definition 2.19 [15]

A soft set (F, E) in a soft topological space (X, τ, E) is said to be soft pre generalized closed (in short soft pg-closed) set, if soft $pcl(F, E) \widetilde{\subseteq} (G, E)$, whenever $(F, E) \widetilde{\subseteq} (G, E)$ and (G, E) is a soft open set in X.

Definition 2.20 [15]

A soft set (F, E) in a soft topological space (X, τ, E) is said to be soft generalized pre closed set (in short soft gp-closed) sets, if soft $cl(F, E) \widetilde{\subseteq} (G, E)$, whenever $(F, E) \widetilde{\subseteq} (G, E)$ and (G, E) is soft Preopen set in X.

Definition 2.21 [16]

A soft set (F, E) in a soft topological space (X, τ, E) is said to be soft generalized β-closed set (in short soft $g\beta$-closed) sets, if soft $\beta cl(F, E) \widetilde{\subseteq} (G, E)$, whenever $(F, E) \widetilde{\subseteq} (G, E)$ and (G, E) is soft open set.

Definition 2.22 [17]

A soft set (F, E) in a soft topological space (X, τ, E) is said to be soft s^*g-closed sets, if soft $cl(F, E) \widetilde{\subseteq} (G, E)$, whenever $(F, E) \widetilde{\subseteq} (G, E)$ and (G, E) is soft semi-open set.

Definition 2.23 [18,4]

A soft set (A, E) in a soft topological space (X, τ, E) is called

1. A soft generalized closed set (Soft g-closed) in a soft topological space (X, τ, E) if $cl(A, E) \widetilde{\subseteq} (U, E)$ whenever $(A, E) \widetilde{\subseteq} (U, E)$ and (U, E) is soft open in (X, τ, E).
2. A soft sg-closed set if $sscl(A, E) \widetilde{\subseteq} (U, E)$, whenever $(A, E) \widetilde{\subseteq} (U, E)$ and (U, E) is soft semi-open.
3. A soft gs-closed set if $sscl(A, E) \widetilde{\subseteq} (U, E)$, whenever $(A, E) \widetilde{\subseteq} (U, E)$ and (U, E) is soft open.
4. A soft rwg-closed set if $cl(Int(A, E)) \widetilde{\subseteq} (U, E)$, whenever $(A, E) \widetilde{\subseteq} (U, E)$ and (U, E) is soft regular-open.
5. A soft wg-closed set if $cl(Int(A, E)) \widetilde{\subseteq} (U, E)$, whenever $(A, E) \widetilde{\subseteq} (U, E)$ and (U, E) is soft open.

Definition 2.24 [19]

A soft set (F, E) in a soft topological space (X, τ, E) is called soft generalized pre regular closed (in short soft gpr-closed) set, if soft $pcl(F, E) \widetilde{\subseteq} (G, E)$ whenever $(F, E) \widetilde{\subseteq} (G, E)$ and (G, E) is soft regular open set.

Definition 2.25 [20]

A soft set (F, E) in a soft topological space (X, τ, E) is called soft regular generalized closed (in short soft rg-closed) set, if soft $cl(F, E) \widetilde{\subseteq} (G, E)$ whenever $(F, E) \widetilde{\subseteq} (G, E)$ and (G, E) is soft regular open set.

Definition 2.26 [21]

A soft set (F, E) in a soft topological space (X, τ, E) is called soft weakly closed (in short soft SW-closed) set, if soft $cl(F, E) \widetilde{\subseteq} (G, E)$ whenever $(F, E) \widetilde{\subseteq} (G, E)$ and (G, E) is soft semi open.

3. SOFT GENERALIZED βb-CLOSED SETS

The present section gives the definition of soft generalized βb-closed set and investigates some of it's properties.

Definition 3.1

A soft set (F, E) in a soft topological space (X, τ, E) is said to be soft generalized βb-closed (in short soft $g\beta b$-closed) set, if soft

$bcl(F,E) \tilde{\subseteq} (G,E)$ whenever $(F,E) \tilde{\subseteq} (G,E)$ and (G,E) is soft β-open. The collection of all soft $g\beta b$-closed sets in (X,τ,E) is denoted by $sg\beta b - C(X)$.

Definition 3.2

A soft set (F,E) in a soft topological space (X,τ,E) is said to be soft generalized b-closed (in short soft gb-closed) set, if soft $bcl(F,E) \tilde{\subseteq} (G,E)$ whenever $(F,E) \tilde{\subseteq} (G,E)$ and (G,E) is soft open set. The collection of all soft gb-closed sets in (X,τ,E) is denoted by $sgb - C(X)$.

Theorem 3.3

Every b-closed soft set in soft topological space (X,τ,E) is soft $g\beta b$–closed.

Proof.

Let (F,E) be a soft b-closed set and (U,E) be any soft β-open such that $(F,E) \tilde{\subseteq} (U,E)$, then $bcl(F,E) = (F,E) \tilde{\subseteq} (U,E)$. Therefore (F,E) is soft $g\beta b$–closed.

Theorem 3.4

A soft set (F,E) is a soft $g\beta b$–closed, iff soft $bcl(F,E) - (F,E)$, does not contain any non-empty soft β–closed sets.

Proof.

Suppose that (F,E) is a soft $g\beta b$-closed and (F_1,E) is a non-empty β-closed soft set such that $(F_1,E) \tilde{\subseteq} bcl(F,E) - (F,E). \Rightarrow$ $(F_1,E) \tilde{\subseteq} bcl(F,E) \tilde{\cap} (F,E)^c. \Rightarrow$ $(F_1,E) \tilde{\subseteq} bcl(F,E)$ and $(F_1,E) \tilde{\subseteq} (F,E)^c$, $(F,E) \tilde{\subseteq} (F_1,E)^c$ since $(F_1,E)^c$ is soft β-open and (FE) is $g\beta b$-closed $bcl(F,E) \tilde{\subseteq} (F_1,E)^c \Rightarrow$ $(F_1,E) \tilde{\subseteq} [bcl(F,E)]^c$. Thus $(F_1,E) \tilde{\subseteq} bcl(F,E) \tilde{\cap} [bcl(F,E)]^c = \Phi$. That is $(F_1,E) = \Phi. \Rightarrow bcl(F,E) - (F,E) = \Phi$ contains no non empty β-closed set.

Conversely: Suppose that soft $bcl(F,E) - (F,E)$, does not contain any non-empty soft β-closed sets, $(F,E) \tilde{\subseteq} (G,E)$ and (G,E) is a soft β-open, if it is possible that $bcl(F,E) \tilde{\nsubseteq} (G,E)$, then $bcl(F,E) \tilde{\cap} (G,E)^c$ is not empty β-closed set of $bcl(F,E) - (F,E)$ which is a contradiction. Therefore, $bcl(F,E) \tilde{\subseteq} (G,E)$. Hence (F,E) is soft $g\beta b$-closed.

Theorem 3.5

If (F,E) is a soft $g\beta b$-closed set, then (F,E) is gb-closed iff $bcl(F,E) - (F,E) = \Phi$ is closed.

Proof.

Assume that (F,E) is soft $g\beta b$-closed. Since $bcl(F,E) = (F,E)$, $bcl(F,E) - (F,E) = \Phi$ is gb-closed and hence closed. Now assume that $bcl(F,E) - (F,E)$ is closed. By known theorem 3.4, $bcl(F,E) - (F,E)$ does not contain any non empty soft β–closed set. That is $bcl(F,E) - (F,E) = \Phi$, thus $bcl(F,E) = (F,E)$. Hence, (F,E) is gb-closed.

Theorem 3.6

If a set (F,E) is soft $g\beta b$ –closed in X then $bcl(F,E) - (F,E)$ contains only null soft closed set.

Proof.

Let (F,E) be a soft $g\beta b$-closed in X and (H,E) be a soft closed set such that $(H,E) \tilde{\subseteq} bcl(F,E) - (F,E)$. Since (H,E) is soft closed its relative complement is soft open, $(H,E) \tilde{\subseteq} (F,E)^c$. Thus $(F,E) \tilde{\subseteq} (H,E)^c$. Consequently $bcl(F,E) \tilde{\subseteq} (H,E)^c$. Therefore, $(H,E) \tilde{\subseteq} \left(bcl(F,E)\right)^c$. Hence $(H,E) = \Phi$ and thus $bcl(F,E) - (F,E)$ contains only null soft closed set.

Lemma 3.7

In a soft topological space we have the following:

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1. Every soft regular open set is soft $g\beta b$-closed.
2. Every soft regular closed set is soft $g\beta b$-closed.
3. Every soft semi-closed set is soft $g\beta b$-closed.
4. Every soft pre-closed set is soft $g\beta b$-closed.
5. Every soft β–closed set is soft $g\beta b$-closed.
6. If (F, E) is soft β-open and soft $g\beta b$-closed then (F, E) is soft b-closed.

Proof.

Obvious.

On Soft Generalized $g\beta b$–closed sets.

Theorem 3.8

If (F, E) is a soft $g\beta b$-closed and (G, E) is a soft β–closed, then $(F, E) \tilde{\cap} (G, E)$ is $g\beta b$-closed.

Proof.

To show that is $g\beta b$-closed, it is enough to show that $bcl(F \cap G, E) \tilde{\subseteq} (U, E)$, where (U, E) is a soft β-open and $(F \cap G, E) \tilde{\subseteq} (U, E)$. Let $(H, E) = (G, E)^c$ then $(F, E) \tilde{\subseteq} (U, E) \tilde{\cup} (H, E)$ since(H, E) is β-open set and (F, E) is $g\beta b$-closed then, $bcl(F, A) \tilde{\subseteq} (U, E) \tilde{\cup} (H, E)$.

Now $bcl(F, A) \tilde{\cap} (G, E) \tilde{\subseteq} [(U, E) \tilde{\cup} (H, E)] \tilde{\cap}$

$(G, E) \tilde{\subseteq} (U, E)$, since every β-closed is b-closed then $bcl(F, E) \tilde{\cap} bcl(G, E) \tilde{\subseteq} (U, E)$ by theorem 3.3 [10], $bcl[(F, E) \tilde{\cap} (G, E)] \tilde{\subseteq} (U, E)$, this implies that $bcl(F \tilde{\cap} G, E) \tilde{\subseteq} (U, E)$. Hence $(F \tilde{\cap} G, E)$ is $g\beta b$-closed set.

Theorem 3.9

If (F, E) is a soft $g\beta b$-closed set in a soft topological space (X, τ, E) and $(F, A) \tilde{\subseteq} (B, E) \tilde{\subseteq} bcl(F, E)$, then (B, E) is a soft $g\beta b$-closed set.

Proof.

Let $(B, E) \tilde{\subseteq} (U, E)$, (U, E) be a soft β-open, then $(F, E) \tilde{\subseteq} (B, E)$ and $(F, E) \tilde{\subseteq} (U, E)$. Since (F, E) is a soft $g\beta b$-closed, $bcl(F, E) \tilde{\subseteq} (U, E)$ since $(B, E) \tilde{\subseteq} bcl(F, E)$, this implies $bcl(B, E) \tilde{\subseteq} bcl(F, E)$. Thus $bcl(B, E) \tilde{\subseteq} bcl(F, E) \tilde{\subseteq} (U, E)$. Therefore, $bcl(B, E) \tilde{\subseteq} (U, E)$, and (U, E) is given soft β-open. Hence, (B, E) is soft $g\beta b$-closed.

Theorem 3.10

If (F, E) is a soft set in soft topological space (X, τ, E), $(F, E) \tilde{\subseteq} (Y, A) \tilde{\subseteq} (X, \tau, E)$ and (F, E) is soft $g\beta b$-closed in (X, τ, E), then (F, E) is soft $g\beta b$-closed relative to (Y, τ_Y, E).

Proof.

Let $(F, E) \tilde{\subseteq} (Y, E) \tilde{\subseteq} (X, \tau, E)$ and (F, E) is soft $g\beta b$-closed set in soft topological space (X, τ, E). To show that (F, E) is a soft $g\beta b$- closed set relative to subspace (Y, τ_Y, E), suppose that $(F, E) \tilde{\subseteq} (Y, E) \tilde{\cap} (U, E)$ where (U, E) is soft β-open set in (X, τ, E), then $bcl(F, E) \tilde{\subseteq} (U, E)$ and bcl$(F, E) \tilde{\cap} (Y, E) \tilde{\subseteq} (U, E) \tilde{\cap} (Y, E)$. Thus, $bcl(F, E) \tilde{\cap} (Y, E)$ is b-closure of (F, E) in (Y, τ_Y, E). Hence, (F, E) is soft $g\beta b$-closed set relative to (Y, τ_Y, E).

Theorem 3.11

If a soft set (F, E) of a soft topological space (X, τ, E) is soft nowhere dense, then it is soft $g\beta b$-closed.

Proof.

Suppose that (F, E) be a soft set nowhere dense and (U, E) be a soft β-open set in (X, τ, E) such that $(F, E) \tilde{\subseteq} (U, E)$,

$bcl(F, E) = sscl(F, E) \tilde{\cap} spcl(F, E) = [(F, E) \tilde{\cup} intcl(F, E)] \tilde{\cap} [(F, E) \tilde{\cup} clint(F, E)]$ since $int(cl(F, E)) = \Phi$. Then $bcl(F, E) = (F, E)$.

Therefore, $bcl(F, E) \tilde{\cup} (U, E)$, which implies that (F, E) is $g\beta b$ –closed in (X, τ, E).

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The converse of the above theorem generally need not be true as shown by the following example.

Example 3.12

Let $X = \{h_1, h_2, h_3, h_4\}$ and $E = \{e_1, e_2\}$ be the set of parameters and $\tau = \{\Phi, X, (F_1, E), (F_2, E)\}$ where $(F_1, E) = \{(e_1, \{h_1, h_3\}), (e_2, \{h_1, h_3\})\}$, $(F_2, E) = \{(e_1, \{h_2, h_4\}), (e_2, \{h_2, h_4\})\}$, so (F_1, E), (F_2, E) are soft open sets and also are soft β-open set, $bcl(F_1, E) = cl(F_1, E) = (F_1, E)$ and $bcl(F_2, E) = cl(F_2, E) = (F_2, E)$, then (F_1, E) and (F_2, E) are soft $g\beta b$–closed sets in (X, τ, E), but not soft nowhere dense.

Theorem 3.13

If a soft set (F, E) of a soft topological space (X, τ, E) is $g\beta b$-closed, then it is gb-closed.

Proof.

Let (F, E) be a soft $g\beta b$–closed set in (X, τ, E) and (U, E) be any soft open set such that $(F, E) \tilde{\subseteq} (U, E)$. Therefore, $bcl\big((A)\big) \tilde{\subseteq} (U, E)$ and (U, E) is a soft open. Thus, (F, E) is a soft gb–closed.

(i) Opposite direction of the above theorem can be achieved if every soft β-open set is soft open set.

(ii) The converse of the above theorem generally need not be true as shown by the following example.

Example 3.14

Let $X = \{h_1, h_2, h_3\}$ and $E = \{e_1, e_2\}$ be the set of parameters with a soft topology $\tau = \{\Phi, X, (F_1, E), (F_2, E)\}$ where

$(F_1, E) = \{(e_1, \{h_1\}), (e_2, \{h_1\})\}$,

$(F_2, E) = \{(e_1, \{h_1, h_3\}), (e_2, \{h_1, h_3\})\}$.

$(F_3, E) = \{(e_1, \{h_1, h_2\}), (e_2, \{h_1, h_2\})\}$ is a soft gb–closed set but not soft $g\beta b$-closed.

Theorem 3.15

Let (F, E) be a soft set of (X, τ, E). If $clint(F, E) \tilde{\subseteq} (U, E)$, whenever $(F, E) \tilde{\subseteq} (U, E)$ and (F, E) be a soft β–open set, then (F, E) is soft $g\beta b$-closed.

Proof.

Assume that $clint(F, E) \tilde{\subseteq} (U, E)$, whenever $(F, E) \tilde{\subseteq} (U, E)$ and (U, E) is a soft β-open subset of (X, τ, E),

$bcl(F, E) =$

$\big[(F, E) \tilde{\cup} \big(clint(F, E) \tilde{\cap}\, intcl(F, E)\big)\big] =$

$[(F, E) \tilde{\cup}\, clint(F, E)] \tilde{\cap}\ [(F, E) \tilde{\cup}\, intcl(F, E)] \tilde{\subseteq}$

$[(F, E) \tilde{\cup}\, clint(F, E)] \tilde{\subseteq}\ [(F, E) \tilde{\cup} (U, E)]$, since $(F, E) \tilde{\subseteq} (U, E)$, then $bcl(F, E) \tilde{\subseteq} (U, E)$. Hence, (F, E) is a soft $g\beta b$ –closed set.

Theorem 3.16

Let (F, E) be a soft set of (X, τ, E). If $intcl(F, E) \tilde{\subseteq} (U, E)$, whenever $(F, E) \tilde{\subseteq} (U, E)$ and (U, E) be a soft β–open set, then (F, E) is soft $g\beta b$-closed.

Proof.

Assume that $intcl(F, E) \tilde{\subseteq} (U, E)$, whenever $(F, E) \tilde{\subseteq} (U, E)$ and (U, E) be a soft β-open set of (X, τ, E),

$bcl(F, E) =$

$\big[(F, E) \tilde{\cup} \big(clint(F, E) \tilde{\cap}\, intcl(F, E)\big)\big] =$

$\big[(F, E) \tilde{\cup} \big(clint(F, E)\big)\big] \tilde{\cap}\ [(F, E) \tilde{\cup}\, intcl(F, E)] \tilde{\subseteq}$

$[(F, E) \tilde{\cup}\, intcl(F, E)] \tilde{\subseteq}\ [(F, E) \tilde{\cup} (U, E)]$.

Since $(F, E) \tilde{\subseteq} (U, E)$, then $bcl(F, E) \tilde{\subseteq} (U, E)$. Hence, (F, E) is a soft $g\beta b$ –closed set.

Theorem 3.17

If a soft set (F, E) in a soft topological space (X, τ, E) is soft s^*g-closed, then it is soft $g\beta b$-closed.

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Proof.

Let (F,E) be a soft $s^{*}g$–closed set and (U,E) be a soft β-open set such that $(F,E)\,\tilde{\subseteq}\,(U,E)$. Since every soft β-open set is soft semi-open, then $cl(F,E)\,\tilde{\subseteq}\,(U,E)$, $bcl(F,E)\,\tilde{\subseteq}\,cl(U,E)$, this implies to $bcl(F,E)\,\tilde{\subseteq}\,(U,E)$ and (U,E) is a soft β-open set. Thus, (F,E) is $g\beta b$-closed.

The converse of the above theorem generally need not be true as shown by the following example.

Example 3.18

Let $X=\{h_1,h_2,h_3,h_4\}$ and $E=\{e_1,e_2\}$ be the set of parameters with a soft topology $\tau=\{\Phi,X,(F_1,E),(F_2,E),(F_3,E)\}$, where $(F_1,E)=\{(e_1,\{h_1\}),(e_2,\{h_1\})\}$, $(F_2,E)=\{(e_1,\{h_2\}),(e_2,\{h_2\})\}$, $(F_3,E)=\{(e_1,\{,h_1,h_2\}),(e_2,\{h_1,h_2\})\}$, the family of all soft closed sets is $\{\Phi,X,(F_1,E)^c,(F_2,E)^c,(F_3,E)^c\}$, where $(F_1,E)^c=\{(e_1,\{h_2,h_3,h_4\}),(e_2,\{h_2,h_3,h_4\})\}$, $(F_2,E)^c=\{(e_1,\{h_1,h_3,h_4\}),(e_2,\{h_1,h_3,h_4\})\}$ and $(_3,)=\{(_1,\{h_3,h_4\}),(_2,\{h_3,h_4\})\}$. $(_1,)$, is a soft open set, soft -open and also soft semi open, $(_1,)\,\tilde{\subseteq}\,(_1,)$ where $(_1,)\,\tilde{\subseteq}\,(_1,)$ but $(_1,)=(_2,)\,\tilde{\nsubseteq}\,(_1,)$. Thus a soft set $(_1,)$ is soft -closed, but not soft -closed.

Theorem 3.19

If a soft set $(,)$ of a soft topological space $(,,)$ is soft –closed, then it is soft –closed.

Proof.

Let $(,)$ be a soft -closed and $(,)\,\tilde{\subseteq}\,(,)$, where $(,)$ is a soft -open set and also soft semi open set, since every soft semi-closed set is soft b-closed set, then $(,)\,\tilde{\subseteq}\,(,)\,\tilde{\subseteq}\,(,)$. Therefore $(,)$ is soft -closed.

The converse of the above theorem generally needs not be true as shown by the following example.

Example 3.20

Let $=\{h_1,h_2,h_3\}$ and $=\{_1,_2\}$ be the set of parameters with a soft topology $=\{\Phi,,(_1,)\}$, $(_1,)=\{(_1,\{h_1,h_2\}),(_2,\{h_1,h_2\})\}$, $(_2,)=\{(_1,\{h_1\}),(_2,\{h_1\})\}$ is soft -closed, but not soft -closed, since $(_2,)\,\tilde{\subseteq}\,(_1,)$, $(_2,)=(_2,)$ but $(_2,)=\tilde{\nsubseteq}\,(_1,)$.

Theorem 3.21

If a soft set $(,)$ of a soft topological space $(,,)$ is soft –closed, then it is soft –closed.

Proof.

Let $(,)$ be a soft -closed and $(,)\,\tilde{\subseteq}\,(,)$, where $(,)$ is a soft -open set and also soft semi open, $(,)\,\tilde{\subseteq}\,(,)$, $(,)\,\tilde{\subseteq}\,(,)\,\tilde{\subseteq}\,(,)$, $(,)\,\tilde{\subseteq}\,(,)$. Therefore $(,)$ is soft -closed.

The converse of the above theorem generally needs not be true as shown by the following example.

Example 3.22

Let $=\{h_1,h_2,h_3\}$ and $=\{_1,_2\}$ be the set of parameters with a soft topology $=\{\Phi,,(_1,)\}$, where $(_1,)=\{(_1,\{h_2,h_3\}),(_2,\{h_2,h_3\})\}$ $(_2,)=\{(_1,\{h_2\}),(_2,\{h_2\})\}$ is soft -closed, but not soft -closed, since $(_2,)\,\tilde{\subseteq}\,(_1,)$, and $(_2,)=(_2,)$, $(_2,)=$.

4. SOFT GENERALIZED –OPEN SETS AND SOFT GENERALIZED –NEIGHBORHOODS

This section introduces the concept of soft generalized –open sets in soft topological space and studies some of their properties.

Definition 4.1

A soft set $(,)$ of a soft topological space $(,,)$ is called a soft generalized –open (briefly soft – open) set, if its complement $(,)$ is soft –closed. The collection of all soft –open sets in $(,,)$ is denoted by $-()$.

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Theorem 4.2

A soft set (,) of a topological space (, ,) is soft -open if and only if (,) $\tilde{\subseteq}$ (,), whenever (,) is a soft -closed and (,) $\tilde{\subseteq}$ (,).

Proof.

Let (,) be soft -open and (,) be a soft -closed contained in (,). Then (,) is soft -closed and (,) is soft -open containing (,), (,) $\tilde{\subseteq}$ (,). Therefore,(,) $\tilde{\subseteq}$ (,).

Conversely: Let (,) $\tilde{\subseteq}$ (,) whenever (,) $\tilde{\subseteq}$ (,) and (,) is a soft -closed. Let (,) be a soft -open set containing (,), then (,) $\tilde{\subseteq}$ (,), (,) $\tilde{\subseteq}$ (,). Hence, (,) is soft -closed. Therefore,(,) is soft -open.

Theorem 4.3

If (,) $\tilde{\subseteq}$ (,) $\tilde{\subseteq}$ (,) and (,) is soft -open then (,) is soft -open.

Proof.

(,) $\tilde{\subseteq}$ (,) $\tilde{\subseteq}$ (,) implies (,) $\tilde{\subseteq}$ (,) $\tilde{\subseteq}$ ((,)) and(,) is soft -closed. Since (,) is soft -closed. Hence (,) is soft -open.

Theorem 4.4

If (,) and (,) are soft -open in X then (,) $\tilde{\cup}$ (,) is also soft -open.

Proof.

Since (,) and (,) are soft -open and by the definition 4.1 their relative complements are soft -closed sets and by theorem 3.12 (,) $\tilde{\cap}$ (,) is soft -closed. Hence (,) $\tilde{\cup}$ (,) is soft -open.

Definition 4.5

A soft set (,) in a soft topological space (, ,) is called a soft -neighborhood of the soft point () $\tilde{\in}$ (, ,) if there exists a soft -open set (,) such that () $\tilde{\in}$ (,) $\tilde{\subseteq}$ (,). A soft set (,) in a soft topological space (, ,) is called a soft -neighborhood of the soft set (,) if there exists a soft -open (,) such that (,) $\tilde{\subseteq}$ (,) $\tilde{\subseteq}$ (,).

A soft -neighborhood generally need not be soft -open as shown by the following example.

Example 4.6

Let = {, }, = $\{{}_1, {}_2\}$ and = {Φ, , $({}_1,)$, $({}_2,)$, $({}_3,)$, $({}_4,)$}, where $({}_1,)$ = {$({}_1, \{\})$, $({}_2, \{, \})$}, $({}_2,)$ = {$({}_1, \{, \})$, $({}_2, \{\})$}, $({}_3,)$ = {$({}_1, \{\})$, $({}_2, \{\Phi\})$}, $({}_4,)$ = {$({}_1, \{\})$, $({}_2, \{\})$} the family of all soft closed sets is {, Φ, , $({}_1,)$, $({}_2,)$, $({}_3,)$, $({}_4,)$} where $({}_1,)$ = {$({}_1, \{\})$, $({}_2, \{\Phi\})$}, $({}_2,)$ = {$({}_1, \Phi)$, $({}_2$, {b})}, $({}_3,)$ = {$({}_1, \{\})$, $({}_2$, {a, b})}$({}_4,)$ = {$({}_1, \{\})$, $({}_2$, {b})} and − () = {Φ, , $({}_1,)$, $({}_2,)$, $({}_3,)$, $({}_4,)$}. A soft set (N, E) such that $(N, E) = \{(e_1, \{b\}), (e_2, \{b\})\}$ is not soft $g\beta b$-open, but it is soft $g\beta b$-neighborhood of a point $G(e_1) = \{b\}$, and also it is soft $g\beta b$-neighborhood of a soft $g\beta b$ –open set (G, E), where $(G, E) = \{(e_1, \{b\}), (e_2, \{\Phi\})\}$. Hence, $(G, E) \tilde{\subseteq} (N, E)$.

Theorem 4.7

Every soft neighborhood is a $g\beta b$ –neighborhood.

Proof.

Let (N, E) be a soft neighborhood of a soft point $F(e) \tilde{\in} (X, \tau, E)$, then there exists a soft open set (G, E)such that $F(e) \tilde{\in} (G, E) \tilde{\subseteq} (N, E)$. As every soft open set (G, E) is a soft $g\beta b$–open set, such that $F(e) \tilde{\in} (G, E) \tilde{\subseteq} (N, E)$. Hence, (N, E) is soft $g\beta b$ –neighborhood of $F(e)$.

Example 4.8

Let $X = \{a, b\}$, and $E = \{e_1, e_2\}$ be a set of parameters with a soft topology $\tau = \{\Phi, X, (F_1, E), (F_2, E), (F_3, E)\}$ $(F_1, E) = \{(e_1, \{a, b\}), (e_2, \{a\})\}$,
$(F_2, E) = \{(e_1, \{b\}), (e_2, \{a, b\})\}$, $(F_3, E) = \{(e_1, \{b\}), (e_2, \{a\})\}$, the family of all soft closed sets is $\{X, \Phi, , (F_1, E)^c, (F_2, E)^c, (F_3, E)^c\}$ where $(F_1, E)^c = \{(e_1, \Phi), (e_2, \{b\})\}$, $(F_2, E)^c = \{(e_1, \{a\}), (e_2, \Phi)\}$,

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$(F_3, E)^c = \{(e_1, \{a\}), (e_2, \{b\})\}$, and soft $\beta - O(X) = \{\Phi, X, (F_1, E), (F_2, E), (F_3, E)\}$. A soft set (N, E) such that $(N, E) = \{(e_1, \Phi), (e_2, \{a, b\})\}$ is soft $g\beta b$-open set and also soft $g\beta b$-neighborhood of a point $G(e_2) = \{b\}$, since $G(e_2) = \{b\} \tilde{\in} (N, E) \tilde{\subseteq} (N, E)$. However, the soft set (N, E) is not a soft neighborhood of the point $G(e_2) = \{b\}$, since (F_2, E) is the only soft open set containing $G(e_2) = \{b\}$ without X and $G(e_2) = \{b\} \tilde{\in} (F_1, E) \tilde{\nsubseteq} (N, E)$.

Theorem 4.9

A soft $g\beta b$-closed set is a soft $g\beta b$-closed neighborhood of each of it's soft points.

Proof.

Let (N, E) be a soft $g\beta b$-closed subset of a soft topological space (X, τ, E) and $Fe \tilde{\in} (N, E)$, it can be claimed that (N, E) is a soft $g\beta b$-closed neighborhood of Fe. (N, E) is a soft $g\beta b$-closed set such that $Fe \tilde{\in} (N, E) \tilde{\subseteq} (N, E)$, since $F(e)$ is an arbitrary point of (N, E), then (N, E) is a $g\beta b$-closed neighborhood of each of it's points.

The converse of the above theorem generally need not be true as shown by the following example.

Example 4.10

Let $= \{a, b, c, d\}$, $E = \{e_1, e_2\}$ be a set of parameters and $\tau = \{\Phi, X, (F_1, E)\}$ is a soft topology over X, where $(F_1, E) = \{(e_1, \{a, b\}), (e_2, \{a, b\})\}$, the family of all soft closed sets is $\{X, \Phi, (F_2, E)\}$, where $(F_2, E) = \{(e_1, \{c, d\}), (e_2, \{c, d\})\}$, the soft set $(F_3, E) = \{(e_1, \{a, b, d\}), (e_2, \{a, b, d\})\}$ is a soft β-open set, $(F_4, E) = \{(e_1, \{a, d\}), (e_2, \{a, d\})\}$, and $(F_5, E) = \{(e_1, \{b, d\}), (e_2, \{b, d\})\}$ are soft $g\beta b$-closed sets, $(F_4, E) \tilde{\subseteq} (F_3, E)$ and $(F_5, E) \tilde{\subseteq} (F_3, E)$, then the soft set $(F_3, E) = \{(e_1, \{a, b, d\}), (e_2, \{a, b, d\})\}$ is a soft $g\beta b$-closed neighborhood of each of its points but $bcl(F_3, E) = X \tilde{\nsubseteq} (F_3, E)$. However, (F_3, E) is not soft $g\beta b$-closed in X.

5. CONCLUSION

In the present, we have introduced soft generalized βb-closed and soft generalized βb-open sets in soft topological spaces which are defined over an initial universe with a fixed set of parameters. We have presented it's fundamental properties with the help of some counter examples. In future these results may be extended to new types of soft generalized closed and open sets in soft topological spaces. And also, In future we will Introduce the fuzzy soft topological space in e-open set [23, 24].

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