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Performance Evaluation of Channel Capacity in MIMO Rayleigh Fading Channels

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Abstract

The demand for Multiple-input Multiple-output (MIMO) system is growing at an explosive rate with the anticipation that communication to a end user anywhere on the globe at all times will be available in the near future .Water-Filling Algorithm (WFA) is presented for MIMO Rayleigh fading environment under Channel Side Information (CSI) is known and unknown at the transmitter. We mainly demonstrate on efficient use of information theoretic Capacity of independently and identically distributed (i.i.d.) MIMO Rayleigh flat fading channels. However, the capacity gain is reduced if the CSI is not perfect. Assuming each antenna in Transmitter is allocated equal amount of power which maximizes capacity. To achieve high capacity gain the reported algorithmcan effectively be optimized for maximizing the channel capacity. We also compared the ergodic channel capacity and channel outage capacity with simulation results. We show that the proposed scheme is spectral efficient, as it offers the full coding rate when the numbers of transmitting and receiving antennas are equal. Moreover the validity of the presented channel estimation algorithm and diversity scheme is verified via computer simulations with MATLAB.

Keywords: MIMO, SISO, LOS, CSI, PDF, Water filing algorithm, i.i.d, SNR.

Introduction

Wireless-system designers are faced with numerous challenges, including limited availability of radio-frequency spectrum and transmission problems caused by s factors such as fading and Multipath distortion. Meanwhile, there is increasing demand for higher data rates, better quality service, fewer dropped calls, and higher network capacity. Meeting these needs requires new techniques that improve spectral efficiency and network links' operational reliability described in [1].MIMO technology promises a cost-effective Way to provide these capabilities. MIMO uses antenna arrays at both the transmitter and receiver.

Single Input Single Output (SISO) channels that send out information over the single paths by using antennas. The Radio signals reflected by the objects, creating multiple Paths that in conventional SISO Channels cause Interference and fading. But MIMO sends data over these multiple paths, thereby increasing the amount of information the system carriers. The data is received Multiple antennas and recombined properly by other MIMO algorithms presented in [2]. Motivated by information theoretic predictions on large spectral efficiency of MIMO systems, recently there has been a great amount of research on various MIMO techniques for wireless communication systems. General N*r*X*Nt* MIMO system is shown figure (1).

Signals in a wireless system frequently reflected by the objects en route to the recipient and bounce along different paths. At various points, the signals become out of synchronization, thereby scrambling the received transmission and decreasing bandwidth, creating a problem called multipath distortion. As Fig1 shows, MIMO takes advantage of this situation by sending a single transmission from two or more antennas to bounce along multiple paths to a receiver. Putting data on multiple signal paths increases the amount of information a system can carry and the number of users it can serve. In addition, this approach lets a system divide a single data set into parts that are sent over multiple paths in parallel. This lets the system handle the information faster than the approaches that send data over a single path reported in [3].

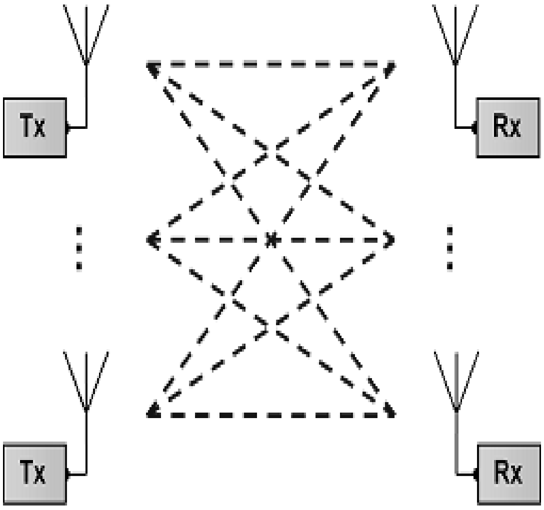

Figure 1.Model of MIMOradio channel

Moreover, by spreading a transmission signal across multiple paths, MIMO increases the chance that any given path will reach the destination, which improves link reliability. In addition, MIMO system allows direct improvement of capacity by simultaneous transmission of multiple data streams. It will showbelow that the information –theoretic capacity for a single link increases linearly with the number of antenna elements presented in [4]. This reduces error rates and improves communication quality reported in [5].

In this paper, we perform multi-user water-filling for a mixed MIMO channel model that includes both Rayleigh fading and shadowing effects. We show that the ergodic capacity and the exact channel outage probability can both be evaluated through numerical WFA. Hence, the time-consuming Monte Carlo simulations, that is, generating a large number of channel realizations and then performing averaging, can be avoided. We also show that for Rayleigh channels without shadowing, multi-user water-filling gains little in capacity over single-user water-filling. For Rayleigh channels with shadowing, multi-user water-filling achieves higher spectral efficiency per antenna over single-user water-filling, with a tradeoff of higher channel outage probability. In either case,single-user water-filling actually has lower computational complexity than multi-user water-filling.

The rest of this correspondence is organized as follows. In Section II, our MIMO channel modelis introduced. In Section III, CSI of transmitter and receiver with mutual information under CSI are derived. The capacity bounds are also derived subject to an average power constraint. In Section IV, optimal power allocation strategies are determined by using water filling algorithm for both ergodic and outage capacities. Finally, numerical results are presented in Section V.

Channel Model

In this we examine fading models for the constructive and destructive addition of different multipath components introduced by the channel. While these multipath effects are captured in the ray-tracing models, in practice deterministic channel models are rarely available, and thus we must characterize multipath channels statistically and we model the multipath channel by a random time-varying impulse response. We will develop a statistical characterization of this channel model and describe its important properties.

If a single pulse is transmitted over a multipath channel the received signal will appear as a pulse train, with each pulse in the train corresponding to the LOS component or a distinct multipath component associated with a distinct scatterer or cluster of scatterers. An important characteristic of a multipath channel is the time delayspread it causes to the received signal. This delay spread equals the time delay between the arrivals of the first received signal component (LOS or multipath) and the last received signal component associated with a single transmitted pulse. If the delay spread is small compared to the inverse of the signal bandwidth, then there is little time spreading in the received signal. However, when the delay spread is relatively large, there is significant time spreading of the received signal which can lead to substantial signal distortion. This is illustrated in[6].In this section we briefly review the capacity formula of MIMO systems. Throughout this paper we assume independent and identically distributed (i.i.d.) Rayleigh flat-fading channel in rich scattering environments, and the channel is unknown at the transmitter and perfectly known at the receiver. Consider a point-to-point communication link and let the numbers of transmit and receive antennas be N*t*andN*r*, respectively. We denote this MIMO communication link as (N*t*, N*r*). ThenNr X 1isthe received signal vector **y** can be determinedas [7]

$$y = Hx + n \quad (1)$$

Where **H** is N*r*X*Nt* channel matrix with the entryhi, hjde scribing the channel gain between the *j*th transmit antenna and the *i*th receive antenna, **x** is N*t*X1 transmit signal vector with independent symbols and **n** is r *1 Additive White Gaussian Noise (AWGN) vector. The entries of **H** are all modeled as i.i.d. circularly symmetric complex Gaussian (CSCG) random variables with variance one, and the AWGN vector **n** satisfies $E\left(nn^{\dagger}\right) = I_r$ in which $n^{\dagger}$ denotes the conjugate transpose of **n** and r **I** denotes $r \times r$ identity matrix. As the channel is unknown at the transmitter, equal power is allocated to each of the transmit antennas. Then the MIMO capacity in bits per second per Hertz (bps/Hz) is derived as

$$C = \log_2 \det \, (2)$$

Where det (i) denotes the determinant operation and ρ is the average SNR at each receive antenna.

Specifically, the capacity is given in terms of the mutual information between the channel input vector **x** and output vector **y** as

$$C = \max_{p(x)} I\left(X;Y\right) = \max_{p(x)} \left[H(Y) - H(Y/X)\right] (3)$$

For H(*Y*) and H(*Y/X*) the entropy in y and y/x as defined. The definition of entropy yields that for H(*Y/X*) = H (N), the entropy in the noise. Since this noise n has fixed entropy independent of the channel input, maximizing mutual information is equivalent to maximizing the entropy in y. The Simplified mutual information is given as

$$\mathrm{I}\left(\mathrm{X};\mathrm{Y}\right) = \mathrm{B}\log_2 \det\left[\mathrm{I}_{\mathrm{M_r}} + \mathrm{H}\mathrm{R}_{\mathrm{x}}\mathrm{H}^{\mathrm{H}}\right] (4)$$

This formula was derived for the mutual information of a multiantenna system, and also appeared in earlier works on MIMO systemsand matrix models for ISI channels.

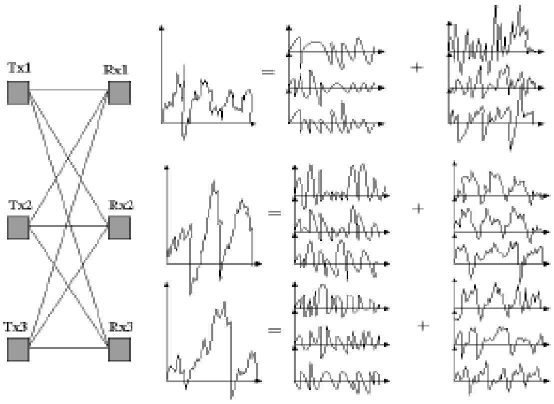

Figure 2.Separation of the received signal into intelligible and non-intelligible components, i.e. signal and noise

Considering However, in non-stationary wireless environments, high complexity on multiple-input-multiple-output (MIMO) channel tracking and large amounts of CSI feedback render such an approach impractical. In this by exploiting the wireless multipath channel structure characterized by the path delays and the path directions-of-departure/ arrival, a new space-time transmit scheme which employs a structure-based water-filling algorithm is proposed.

New Approach for MIMO Capacity for Rayleigh channel Model with CSI at Receiver

In previous sections we have argued that knowing the channel can provide various performance boosts for MIMO systems and potentially enable applications like perfect wireless secrecy illustrated in[8] which are difficult if not possible without good channel information. These potential benefits prompt us to ask just how difficult channel identification and tracking is at a fundamental information theoretic level. To answer such questions requires some sort of channel model. In a typical depiction, a MIMO channel is shown as a cloud in which something called .scatteringOccurs. This scattering is usually summarized by a stochastic sequence of channel matrices Gi. A large amount of work has been devoted to characterizing this stochastic process in a variety of situations with the most prevalent being complex zero mean circularly symmetric (ZMSW)Gaussian channel gains with or without correlation between entries presented in [9]. The information theoretic capacity of MIMO channel with different assumptions described in [10] Owing to the previously discussed strong improvements possible when channel state is known, a large amount of work has also been devoted to estimating MIMO channel matrices reported in[11, 12], and references therein). In work roughly similar to the approach we will take, a "parametric" method described in[13] of modeling and estimation was used with specified number of rays along with separate delay and phase estimates since delay profiles vary much more slowly than phase for exactly the reasons reported in [14] macro based objects moves very slowly.

Channel Known to Transmitter

We analyze the transmitter with Channel Side Information at Transmitter (CSIT) and Channel Side Information at Receiver) (CSIR)the transmitter optimizes its transmission strategy for each fading channel realization as in the case of a static channel. A short-term power constraint assumes that the power associated with each channel realization must be equal to the average power constraint P. In this case the ergodic capacity becomes

$$C = E_H\left[\max_{\rho_i:\sum_i \rho_i \le P} \sum_i B\log_2\left(1+\frac{P_i\gamma_i}{P}\right)\right] \quad (5)$$

A less restrictive constraint is a long-termpower constraint, where we can use different powers for different channel Realizations subject to the average power constraint over all channel realizations.

Channel Unknown at Transmitter

Consider now a time-varying channel with random matrix **H** known at the receiver but not at the transmitter. The transmitter assumes a ZMSW distribution for **H**.

The two relevant capacity definitions in this case are ergodic capacity and capacity with outage. Ergodic capacity defines the maximum rate, averaged over all channel realizations whereas capacity with outage represents to quantify how often the transmission is blocked with some probability error that can be transmitted over the channel for a transmission strategy based only on the distribution of **H**.

$$C = \max_{R_X : Tr(R_X)=\rho} E_H \left[B \log_2 \det \left[I_{M_r} + H R_X H^H \right] \right] \quad (6)$$

Where the expectation is with respect to the distribution on the channel matrix, which for the ZMSW model isi.i.d. zero-mean circularly symmetric unit variance.

Water filling Algorithm

In this we demonstrated the MIMO channel capacity better than SISO channel capacityand to achieve high capacity gain another method is water filling concept is proposed. In this concept it can also happen that some sub channels that have a poor SNR, do not get any power assigned. Water filling makes sure that energy is not wasted on sub channels that have poor SNR: in the OFDM (Orthogonal Frequency Division Multiplexing) context this means not wasting power on subcarriers that are in a deep fade.

With water filling, power is allocated preferably to sub channels that have a good SNR. This is optimum from the point of view of theoretical capacity; however, it requires that the transmitter can actually make use of the large capacity on good sub channels.

CSI must be known to transmitter than considering the antennas i.e., which antennas giving better SNR rate, those ant., given much essential power to get the high data rate. Considering single-user and multi user which is described below.

Single-User Water filling

With regard to the Waterfilling theorem, the channel is completely characterized by its channel gain to noise ratio (CNR), defined as

$$T_a = \frac{H\,|_n|^2}{\Gamma \cdot \sigma_n^2}, n = 1, 2, ..., N \quad (7)$$

In this equation, we already incorporated the SNR gap presented in [15], which accounts for the desired symbol error ratio (SER), assuming QAM modulation. For the classical Waterfilling theorem which maximizes the channel capacity Γ = 1 has to be chosen. Then, according to the Waterfilling theorem, the power on sub channel *n* is given by

$$E_n = [C_o - T_n^{-1}]^+, where[x]^+ = x \text{ for } x > 0, \text{ else } 0 (8)$$

and the "water level" C_o must be chosen such that

$$E_{tot} = \sum_{n=1}^{N} E_n \quad (9)$$

Eq. (4), (5) can be visualized with the Waterfilling diagram shown in Fig. 4.Symbol sequence on sub channel *n* is given by

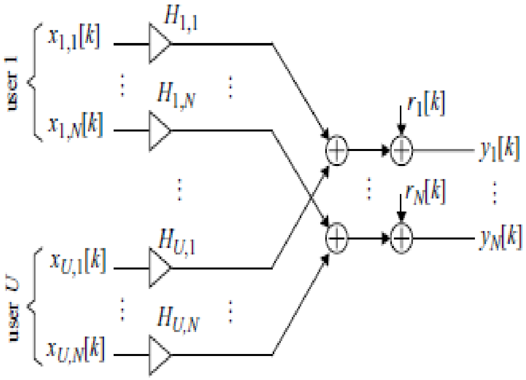

Figure 3.Channelmodel for multiuser MIMO-OFDM

Multiuser Water filling

In the multiple access channel shown in Fig. 3 the received symbol sequence on sub channel *n* is represented by the combination of channel impulse response and noise. The resulting received symbol sequence is presented as Eq.(10) $y_n[k] = \sum_{u=1}^{U} H_{u,n} \cdot x_{u,n}[k] + r_n[k]$ (10)

and the CNRs for multi-user water filling is defined by

$$T_{u,n} = \frac{|H_{u,n}|^2}{\tau_u \times \sigma_n^2} \quad (11)$$

The generalization of the Waterfilling theorem involves the idea of an equivalent channel, $\hat{H}_{u,n} = H_{u,n}/\sqrt{b_u}$ which leads to the adoption of the equivalent transmit power. $\hat{E}_{u,n} = b_u \times E_{u,n}$.

This approach makes it possible to combine[16]

Multiuser systems with multiple antennas at the transmitter(s) and/or receiver(s) are called MIMO multiuser systems. These multiple antennas can significantly enhance performance in multiple ways. The antennas can be usedto provide diversity gain to improve BER performance. The capacity region of the multiuser channel is increased by MIMO, providing multiplexing gain. Finally, multiple antennas can provide directivity gain to spatially separate a user, which reduces interference. There is typically a tradeoff between these three types of gains in MIMO multiuser systems [17].

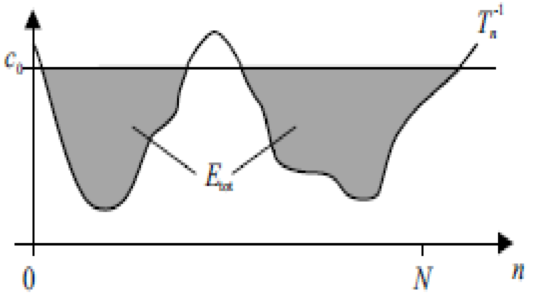

Figure 4.Single user Waterfilling algorithm

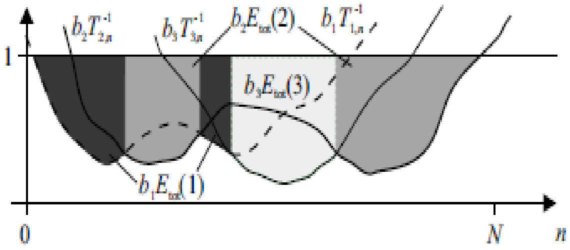

Figure 5.Multi-User Waterfilling algorithm

The Water filling diagrams of different users: for each user, the multiplier ‘bu’ is chosen such that the water level is unity. Then, the individual diagrams can be combined to one as shown in Fig. 5.

The power constraint of user u is denoted by Emax (u):

$$E_{tot}(u) = \sum_{n=1}^{N} E_{u,n} \le E_{\max}(u) \quad (12)$$

Note, that for the case that a total power constraint is given instead of user-individual power constraints, the subcarrier allocation becomes a trivial task: for each sub channel the user with the highest CNR is chosen.

Results and Discussion

In this paper, a study of MIMO system with partial or imperfect CSI at the transmitter has been presented for the MIMO capacity for spatial channel model with partial CSI knowledge. In the paper, MIMO channel capacity based on statisticsCSI has been established using Mat lab software, this paper presented a general framework based on replica method to discuss the channel estimation and achieve of channel capacity. Our analyses show that the channel capacity depends on the accuracy of the channel estimation, the system load and signal-to-noise ratio. Clearly, the channel capacity increases with signal-tonoiseratio. From below figure we can observe that each channel is identically independently distributed as no four channels are same propagation.

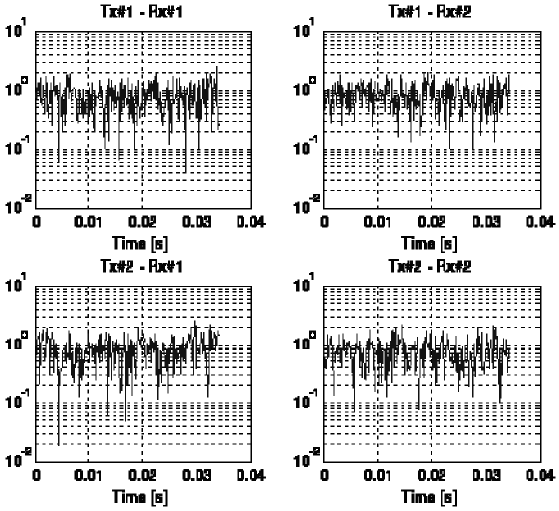

Figure 6.Fading Characteristic Curves of Channel MatricesH

The Fading distribution of Channel MatricesH is related to the propagation environment, and fading characteristic curves of channel matrices are as shown in Fig. 6. There are four resolved paths in the curves and the top curve is the first path which is LOS with. Nt=Nr= 2 .To examine the fading characteristics of the proposed MIMO channel model in space-time-frequency dimensionality, the ergodic channel capacity , outage capacity and signal to noise ration are given as following Fig. 7, Fig. 8, Fig.9 and Fig.10.

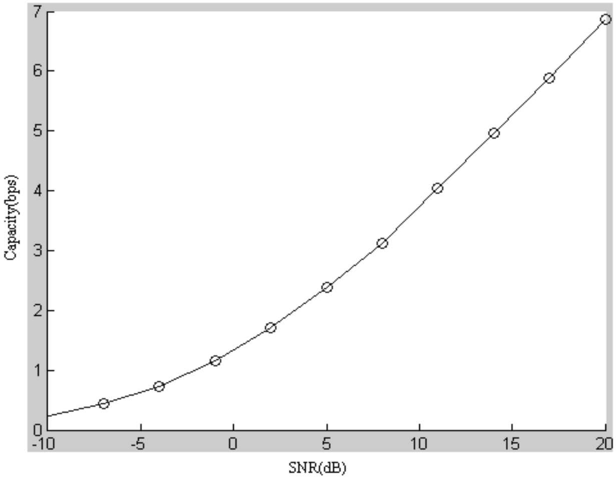

Figure 7.Capacity Vs SNR for Nt=1 and Nr=1

Figure 7showsCapacity versus SNR for Nt=1 antenna at transmitter and Nr=1 antennaat Receiver. The channel capacity is up to 7b/s/Hz with signal to noise ratio 20 dB.

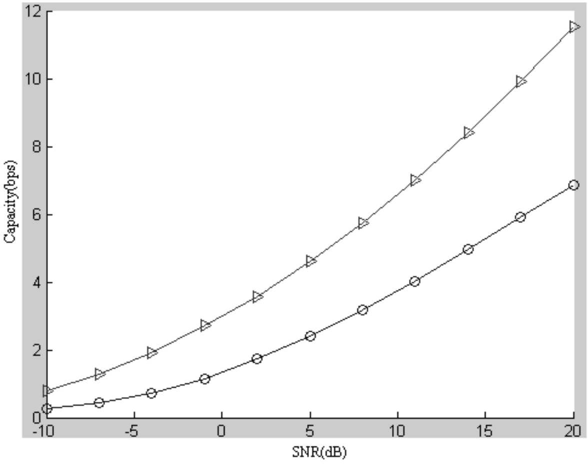

Figure 8.Capacity Vs SNR for Nt=2 and Nr=2

Figure.8 represents Capacity versus SNR for Nt=2 antennasat transmitter and Nr=2 antennasat Receiver. The estimated channel capacity is up to 12 b/s/Hz.

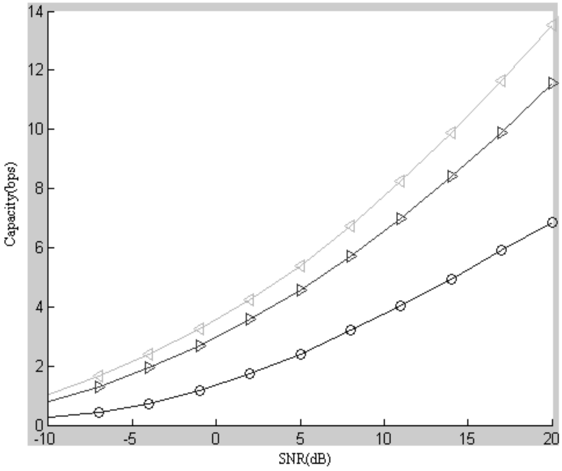

Figure 9.Capacity Vs SNR for Nt=3 and Nr=2

Figure.8 represents Capacity versus SNR for Nt=3 antennas at transmitter and Nr=2 antennas at Receiver. The estimated channel capacity is up to 14 b/s/Hz.

TABLE 1.Channel Capacity Value by Nt and Nr.

S.NO.	NT --NR	CHANNEL CAPACITY
1	1-1	7b/s/Hz
2	2-2	10b/s/Hz
3	3-2	14b/s/Hz
4	4-4	22b/s/Hz

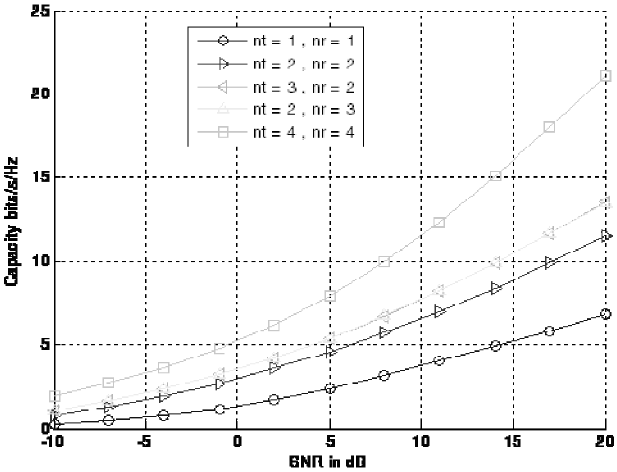

Figure 10.Capacity Vs SNR for Nt=4 and Nr=4

Figure 10 shows the performance comparison shows the performance comparison of ergodic capacity of a MIMO channel with Nt= Nr=4 when the channel state information is unknown to the transmitter and also known, the channel is Rayleigh i.i.d [18]. Here the capacity of MIMO system for spatial channel model is increasing with the SNR and as the number of transmitters and receiver are being increased, also the capacity increases (for Nt=4 antenna transmitter and Nr=4 antenna Receiver)the channel capacity is up to 22b/s/Hz.

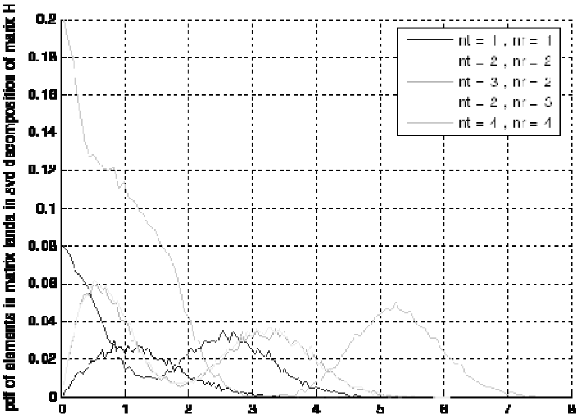

Figure 11.Comparisons of MIMO Channels with PDFs

We knowthat multiple antennas at the transmitter or receiver can be used for diversity gain. When both the transmitter and receiver have multiple antennas, there is another mechanism for performance gain called multiplexing gain.

The multiplexing gain of a MIMO system results from the fact that a MIMO channel canbe decomposed into a number R of parallel independent channels. By multiplexing independent data onto these independent channels, we get an R-fold increase in data rate in comparison to a system with just one antenna atthe transmitter and receiver. This increased data rate is called the multiplexing gain. In this above figure 11 can observe independent channels from a MIMO system.

Figure 12 gives the comparison of Ergodic (Shannon) capacity and outage capacity with signal to noise ratio. Where Ergodic value means the average of maximum number of capacity gain and outage value is while in data transmission effects causes the loss of data in the receiver .these can be characterized by two cases channel known to transmitter and channel unknown and receiver feedback also considered. Figure 12 shows the ergodic capacity and outage capacity over different system configurations as a function of γ. Note that, the ergodic capacity increases with increasing γ and with increasingnumber of transmitting antennas Nt and number of receiving antennas Nr.

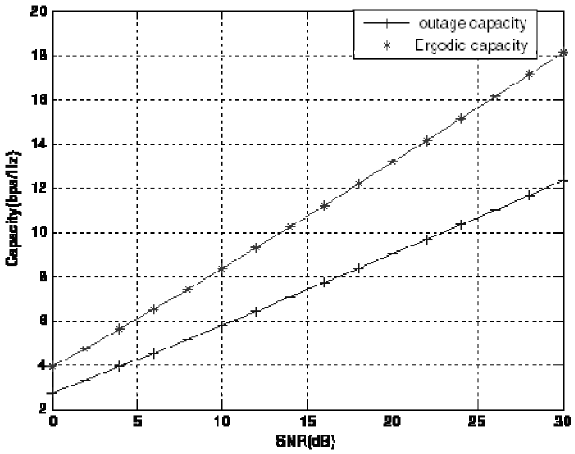

Figure 12.Comparison of Ergodic and Outage Capacities with CSI at receiver

Note that, the ergodic capacity when the channel state Information is known to the transmitter is always higher than the ergodic capacity when it is unknown. This advantage reduces at high SNRs. This is can be explain as follows. At high SNRs, the water filling principle solution will approach the average power allocation. From Eq. (5), the capacity of the MIMO tends to Eq. (6). The ergodic capacity is optimized which is always greater than the outage capacity of the MIMO channel.

Conclusion

In this paper the capacity of MIMO fading channel under Rayleigh environment has been investigated. Our analyses show that the channel capacity depends on the accuracy of the channel estimation, and signal-to-noise ratio. This paper introduced the study of MIMO system with partial or imperfect CSI at the transmitter has been presented for the MIMO capacity for spatial channel model with partial CSI knowledge. This paper presented a general framework based on replica method to discuss the impact of imperfect channel estimation error on multi-user channel capacity. However, increasing signal-to-noise ratio is helpful to improve the capacity. Particularly, in practice, imperfect channel state information decreases the channel capacity of multi-user MIMO channel when system capacity and signal-to-noise ratio are specified. In other words, when the variance value of channel estimation error increases, the efficient transmitting rate of the system is attenuated. And the degree of capacity loss due to imperfect channel state information is obtained. Water filling algorithm not only determines the maximizing sum capacity value, but also the adaptive transmit policies that achieve optimum. This paper concludes by providing numerical results which demonstrate that the algorithm takes very little iteration to converge to the optimum.

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