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Minimizing Mathematical Benchmark Test Functions using Selection Based Particle Swarm Optimization

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Abstract

This article is based on a new technique called selection based particle swarm optimization (SBPSO) which has been developed and implemented to mathematical benchmark functions. In SBPSO, the numbers of particles are decreased in each iteration by decrement factor ‘t’. Elimination of particles is based on their function value. SBPSO is used as a guideline for performance testing by different functions. The benchmarks we used are Rosen brock, Beal and booth. We implemented the original algorithm and obtained optimized results for every function. The results of SBPSO have been compared with that of BPSO, and SBPSO is found to converge faster with more accurate results. SBPSO is compared with BPSO by the number of times the objective function has been processed (counted) in optimization technique.

Keywords: Mathematical benchmark function, Selection based particle swarm optimization, Basic particle swarm optimization, Decrement factor.

Introduction

For many years particle swarm optimization has emerged as a new technique for global optimization. Many researchers and scientists have aggrandized the primary idea with amendment ranging from inconsequential parameter adjustments to complete replication of the algorithm, while many others are working just on the comparison and resemblance with the other optimization algorithm family [1]. The algorithm was first developed by Eberhart and Kennedy in 1995. As proposed by Eberhart and Kennedy, the PSO algorithm is a flexible algorithm depending on a socio-psychological allegory, a population of individuals (referred to as particles) acclimated by returning stochastically toward previously successful regions [2]. Particle swarm optimization (PSO) plays a vital role in finding solutions for continuous optimization problems and also acts as an alternative for global optimization. The designing of a standard PSO is which has been taken into account by the latest research and developments, and is used as a guideline for performance testing by different functions [5].

It is understood that the PSO methods can give a top-notch arrangement with basic implementation and quick convergence. PSO algorithm has been produced for the nonlinear consistent optimization issue to accomplish the best compromise arrangement. In this work, the expense of generation is taken as the objective which should have been minimized. Adjustments in PSO have been indicated in some recent research papers. These changes make PSO algorithm demonstrate more preferable result over the basic PSO. In this thesis, work modifications have been done in PSO which is named as MPSO (modified particle swarm optimization). This change is done to enhance the speed and it gives better results.

Wang et al. [3] explained the ant-system algorithm-based Rosenbrock function optimization in multi-dimension space. Marco et al. [4] explained the effectiveness of the imperialist competitive algorithm-a socio-political-inspired algorithm-on finding the optimal solution for different kinds of minimization functions as well as different kinds of landscapes.

In this article, a selection based particle swarm optimization (SBPSO) technique for solving the mathematical benchmark function is proposed. The feasibility of the proposed method has been demonstrated for Rosenbrock, Beal, and booth. The results have been compared with basic PSO and the results indicate faster convergence and are found to be more accurate. The particles are eliminated based on their function value. The minimum number of particles ‘q’ have been fixed to 5 or 10 in case the number of particles become less than ‘5’ or ‘10’ in any iteration. SBPSO is then run as BPSO with ‘q’ numbers of particles.

Basic Particle Swarm Optimization

PSO, first developed by Eberhart and Kennedy (1995) [7], is a population-based optimization algorithm. PSO has established in two fundamental segment strategies. Maybe more clear are its binds to artificial life and to bird flocking, fish schooling, and swarming hypothesis specifically. It is likewise related, be that as it may, to evolutionary computation, and has binds to both genetic algorithms and evolution strategies. In PSO, the population is called ‘swarm.’ Each potential solution is called particle which is given a random velocity and is flown through the solution space (similar to the search process for food of a bird swarm) searching for the optimal position. The PSO idea comprises, at every single time step, changing the velocity (accelerating) every particle toward its *pbest* and *gbest* areas (global adaptation of PSO). Each particle keeps track of its previous best position, called pbest, and corresponding fitness in its memory. The best value of pbest is called gbest, which is the best position discovered by the swarm.

The modified velocity and position of each particle are calculated as given below.

Velocity-updating equation:

$$V_i^{k+1} = W \times V_i^{k} + C_1 \times rand_1 \times (Xpbest_i - X_i^{k}) + C_2 \times rand_2 \times (Xgbest - X_i^{k}) \quad \ldots(1)$$

Position-updating equation:

$$X_i^{k+1} = X_i^{k} + V_i^{k+1} \quad \ldots(2)$$

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Where:

V_i^k: velocity of agent i at iteration k,

W: inertia weight factor,

C_j: acceleration constant,

$rand_j$: uniformly distributed random number between 0 and 1,

X_i^k: current position of agent *i* at iteration *k*,

$Xpbest_i$: pbest of agent itself,

Xgbest: gbest of the group.

Selection Based Particle Swarm Optimization

In SBPSO, modification has been done in a basic particle swarm optimization (BPSO) method. In BPSO, numbers of particles are specified and they remain same for all the iterations in the algorithm. In SBPSO, a different method for selecting the number of particles is used. The number of particles is specified before an optimization problem at random in the zeroth iteration. In SBPSO the number of particles is decreased in each iteration by some decrement factor ‘t’. The number of particles decrease to the ‛t’times of value in every iteration. For example, if the size of particles is taken as 160 and ‘t’ is 2, then in each iteration size of the particles will decrease two times of the size of particles in previous iteration. In the first iteration there will be 160 particles, in the second iteration it will decrease two times and become 80, in third iteration it will become 40 and so on. This decrement of particles in each iteration is being done in a way that particles with lesser function value will be selected and with higher function value will get discarded. So in each iteration, the size of particles changes by the factor ‛t’. The particles are sorted according to their function value and the size found by the factor ‛t’ will select the top-sorted particles (based on minimum function value) for the subsequent iterations and this procedure will be repeated in every iteration. In SBPSO, the particles decrease in each iteration by $\frac{p}{t}$ and they may become a fraction or less than minimum number of particles required to optimize a function. In that case, the number of particles is fixed to ‘q’ which is the minimum number of particles required to optimize a function.

The value of random numbers Rp and Rg have been kept to fix values 0.5 and 0.6 respectively and the values of constriction factor Cp and Cg have been fixed to 2 in velocity-modification equation.

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Mathematical Benchmark Test Functions Using SBPSO

Artificial landscapes, the second name given to the test functions, are very useful to evaluate characteristics of optimization algorithms. In this case of application of SBPSO to the mathematical benchmark functions, the SBPSO algorithm can be applied directly to the particular mathematical function, i.e., without any modification. As the mathematical functions are single-objective functions and no equality criteria on the fitness functions values, no further formulation for objective function is required and the inequality constraints on the variables, if present, are taken care of in the SBPSO algorithm itself.

Here, SBPSO for the optimization of some mathematical benchmark functions, are as follows:

(i). Rosenbrock's function

$f(X_1,X_2)=100*(X_2-X_1^{2})^{2}+(X_1-1)^{2}$ ....(3)

(ii) .Booth's function

$f(X_1,X_2) = (X_1 + 2 * X_2-7)^{2} + (2 * X_1 + X_2-5)^{2}$ ...(4)

(iii).Beale's function

$f(X_1,X_2) =(1.5-X_1+X_1*X_2)^{2} + (2.25-X_1+X_1*X_2^{2})^{2} + (2.625-X_1+X_1*X_2^{3})^{2}$ ......(5)

Where X_1 and X_2 are variables.

Computational Results

SBPSO has been applied to mathematical benchmark test function and results are shown in Tables 1, 2 and 3 respectively.

Table 1.Results of Rosenbrock Function by SBPSO Varying p, t and q

S. No.	p	t	Q	F	Count	Iterations
1*	10	1	10	-19	1210	121
2	20	2	10	-16	1150	112
3	40	2	10	-14	1050	97
4	80	2	10	-21	1210	102
5	80	4	10	-22	1200	104
6	160	2	10	-15	1110	69
7	160	4	10	-16	1170	83
8	320	4	10	-21	1640	93
9	40	2	5	-9	550	91
10	80	2	5	-10	675	93
11	80	4	5	-6	630	92
12	160	2	5	-11	885	88
13	160	4	5	-10	805	90
14	320	4	5	-12	1100	75
15	320	8	5	-7	1080	82

Table 2.Results of Beales Function by SBPSO Varying p, t and q

S. No.	p	t	Q	F	Count	Iteration
1*	10	1	10	-12	1250	124
2	20	2	10	-10	980	94
3	40	2	10	-12	1090	100
4	80	2	10	-10	1150	95
5	80	4	10	-20	940	77
6	160	2	10	-19	1300	87
7	160	4	10	-20	1210	86
8	320	4	10	-8	1300	74
9	20	2	5	-9	450	81
10	40	2	5	-17	490	75
11	80	2	5	-8	655	88
12	80	4	5	-10	575	80
13	160	4	5	-7	775	83
14	320	4	5	-8	1100	74
15	320	8	5	-14	585	62

Table 3.Results of Booth Function by SBPSO Varying p, t and q

S. No.	P	T	q	F	Count	Iteration
1*	10	1	10	−19	1300	129
2	20	2	10	−17	1020	99
3	40	2	10	−19	950	87
4	80	2	10	−18	1130	94
5	80	4	10	−15	1130	97
6	160	2	10	−18	1280	86
7	160	4	10	−14	1230	89
8	320	4	10	−16	1540	88
9	20	2	5	−12	505	93
10	40	2	5	−12	495	80
11	80	2	5	−11	640	86
12	80	4	5	−8	695	105
13	160	2	5	−10	855	82
14	160	4	5	−8	755	80
15	320	2	5	−13	1205	57
16	320	4	5	−11	1110	79
17	320	8	5	−16	1070	80

In Tables 1, 2 and 3, the S. No. 1* is the result of BPSO. The number of counts is significantly less than BPSO for the particle size 20, 40, 80, 160 and 320 with the decrement factor ‘t’ as 2, 4 and 8 for the minimum number of 10 particles. Similarly, the number of counts is decreasing for the particles 20, 40, 80, 160 and 320 for the decrement factor of 2, 4, 8 and 16 for the minimum number of five particles. The benchmark test functions are found to converge faster and are more accurate for the combination of parameters shown in Table 4 for Rosenbrock, Beale and Booth.

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Table 4.Comparison of BPSO and SBPSO Result

S. No.	Function	BPSO Count	SBPSO				% Saving of Count
			p	t	q	Count	
1.	Rosenbrock	1210	80	2	5	675	44.21
2.	Beale	1250	80	2	5	575	54
3.	Booth	1300	80	2	5	640	50.76

Conclusion

SBPSO has been successfully applied to mathematical benchmark functions-Rosenbrock, Beale and Booth function. Selection based PSO (SBPSO) has been developed in which a better population of particles is selected in each iteration based on function value. Size of particles in each iteration is decreased by decrement factor ‘t’. SBPSO is found to converge for minimum ‘5’ particles for which basic PSO fails to converge. SBPSO is found to be computationally faster than BPSO. With large increase in size of particles and decrement factor, results are found to be more accurate but the number of counts increases.

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