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Designing of Quadrature Mirror filter Bank

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Abstract

Quadrature mirror filter (QMF) banks have been of great interest during the past decade. These filters find application where a discrete signal is to split into a number of consecutive bands in the frequency domain, so that sub-band signal can be processed in an independent manner and sufficient compression may be achieved. Typical processing includes under sampling the sub-band signals, encoding them and transmitting over a channel. Eventually, at some point in the process, the sub-band signals should be recombined so that original signal is properly reconstructed. Typical application of such signal splitting include sub-band coders for video signals [2], digital trans-multiplexers used in FDM/TDM conversion, and frequency domain speech scramblers. In this thesis we look at the QMF design problem purely as a signal reconstruction problem. The channel is therefore assumed to be noiseless and the exact signal characteristics are not given. We treat the QMF design problem as a multivariable optimization over the filter coefficient.

Polyphase Structures & Polyphase decomposition

The process of converting a signal from a given rate to a different rate is called sampling rate conversion. Systems that employ multiple sampling rates in the processing of digital signals are called multirate digital processing systems. The efficiency of digital filter can be increased by reducing the large FIR filter of length M in to a set of smaller filters of length K= M/I where M is selected to be a multiple of I. Since the up-sampling process inserts I-1 zeros between successive values of x (n), only K out of M input values stored in the FIR filter at any one time are non-zero. At one time instant these non-zero values coincide and are multiplied by the filter coefficient h (0), h (I), h (2I),……h (M-I). This observation leads to define a set of smaller filters called polyphase filters with unit sample responses.

$P_k(n) = h\,(k+nI)$ k= 0,1,……I-1 n= 0,1,……K-1(1.1)

Where K = M/I is an integer

The polyphase sub-filters are basically all-pass filters and differ primarily in their phase characteristics, this explain the reason for the term **Polyphase**. The polyphase filter can be viewed as a set of I sub filters connected to a common delay line.

Polyphase QMF Banks

The QMF banks can be realized efficiently if polyphase structures are used. We start with the two channel analysis filter bank and with the equations $H_0(z) = H(z)$ and $H1(z) = H(-z)$ for the filter transfer functions. Representing the low pass and high pass transfer functions in polyphase form:

$$H_0 (z) = H_{00} (z^2) + z\text{-}^1 H_{01} (z^2) \tag{1.2}$$

$$H_0 (z) = H_{00} (z^2) - z\text{-}^1 H_{01} (z^2) \tag{1.3}$$

Since $H_1 (z) = H_0 (-z)$

V_0 (z) Low frequency component

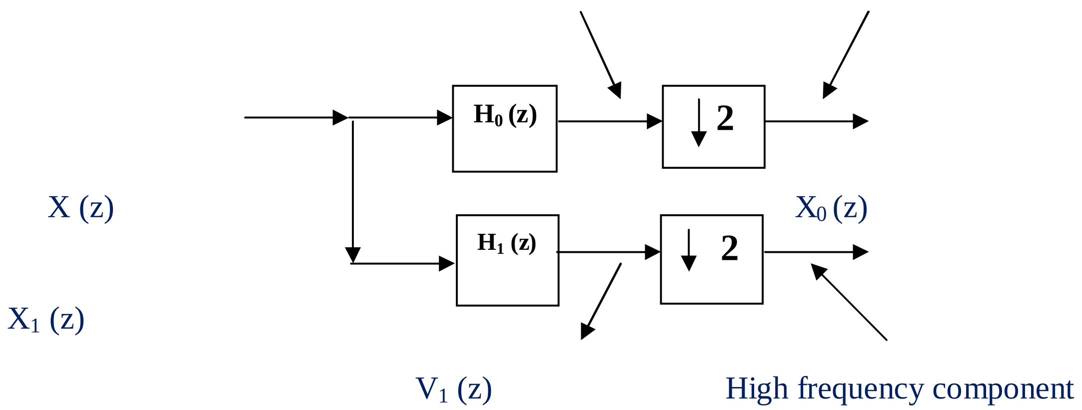

Figure 1.1.Two channel analysis filter

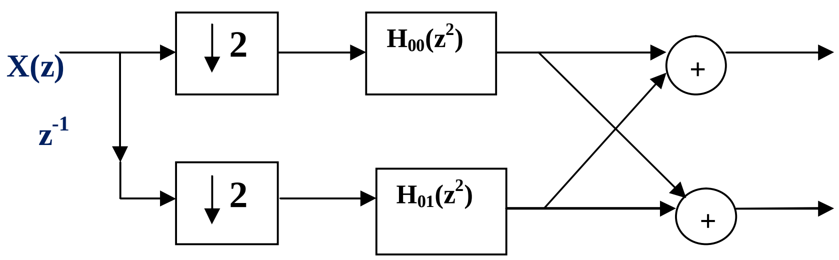

Figure 1.2.Two channel polyphase analysis filter

Both channels have common polyphase components in their transfer function. An analysis of the configuration in fig 3.1 results in:

$$V_0 (z) = H_{00} (z^2).\ X (z) + z^{-1}\ H_{01} (z^2).\ X (z)$$

$$= H_0 (z).X (z) \tag{1.4}$$

$$V_1 (z) = H_{00} (z^2).\ X (z) - z^{-1}\ H_{01} (z^2).\ X (z)$$

$$= H_1\ (z).X\ (z) \tag{1.5}$$

Finally, two down-sampler with factor 2 follow at the end of analysis filter bank. They can be placed before the filter, in accordance with the third party as in fig 1.2 . In this manner, half as many filter operations per seconds as before need to be performed. With the sampling rate and the number of operations halved the overall gain is 4:1.

The synthesis filter bank consists of a low-pass filter G_0 (z) and high-pass filter G_1 (z), which is associated by the relationship G_1 (z) = -G_0 (-z), if we expand G_0 (z) and G_1 (z) to its polyphase form:

$$G_0\ (z) = G_{00}\ (z^2) + z^{-1}G_{01}\ (z^2) \tag{1.6}$$

$$G_1\ (z) = -G_{00}\ (z^2) + z^{-1}G_{01}\ (z^2) \tag{1.7}$$

So we can written it as :

$$Y\ (z) = G_{00}\ (z^2).\ [X_0(z) +X_1(z)]+z^{-1}G_{01}(z^2).\ [X_0(z) +X1(z)]= G_0\ (z).X_0\ (z) +G_1\ (z).X_1\ (z) \tag{1.8}$$

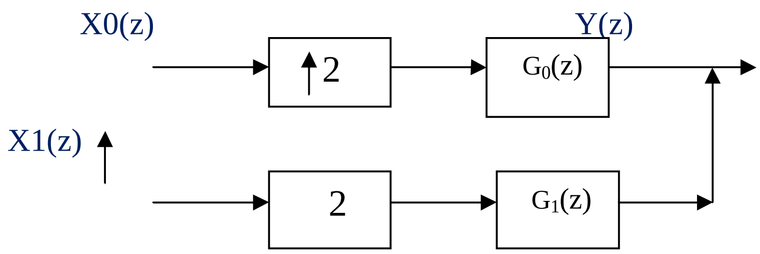

Figure 1.3.Two channel synthesis filter

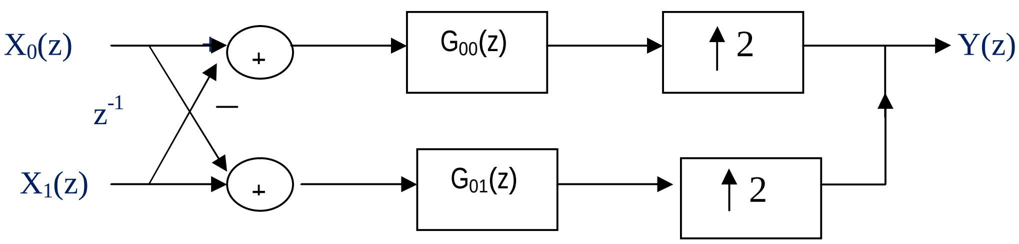

Figure 1.4 Two channel polyphase synthesis filter

Using the sixth identity, the up-samplers at the input of the synthesis filter bank can be moved to the output. As a result we obtain the computational complexity that is reduced by a factor of 4 compared to original version.

General Two Channel Polyphase Filter Banks

In the following, topological aspects of two channel filter banks will be treated rather than characteristics of transfer functions. The following considerations are focused on the two-channel

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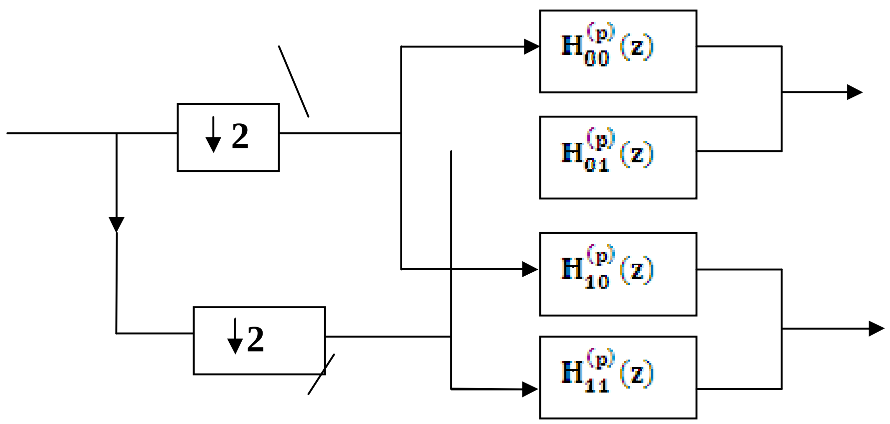
analysis filter bank. By decomposing the two transfer functions H_0 (z) and H_1 (z) into their polyphase components as:

Ho (z) = $H_{oo}^{(p)}(z^2) + z^{-1}H_{o1}^{(p)}(z^2)$,

H_1 (z) = $H_{1o}^{(p)}(z^2) + z^{-1}H_{11}^{(p)}(z^2)$, (1.9)



The polyphase analysis filter consists of two parts, an input de-multiplexer and a discrete time system with two inputs and two outputs, which is operated at the reduced sampling rate. The system is defined by the vector equation:

$$\begin{bmatrix} X_0(z) \\ X_1(z) \end{bmatrix} = \mathbf{H}^{(p)}(z) \cdot \begin{bmatrix} X_{0E}(z) \\ X_{1E}(z) \end{bmatrix} \qquad (1.10)$$

Where

$$\mathbf{H}^{(p)}(z) = \begin{bmatrix} H_{oo}^{(p)}(z) & H_{o1}^{(p)}(z) \\ H_{10}^{(p)}(z) & H_{11}^{(p)}(z) \end{bmatrix} \qquad (1.11)$$

Diagrammatically it can be shown as:

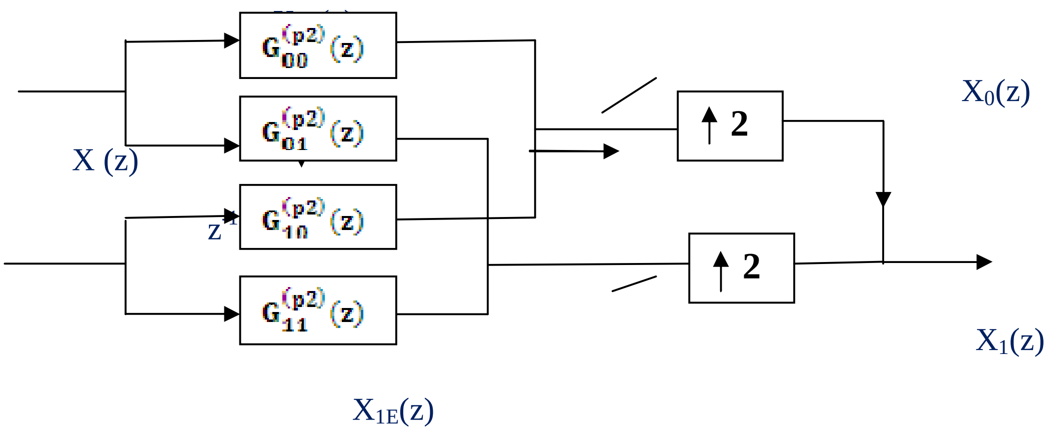


Figure 1.5.Polyphase analysis filter bank

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Again, we start from two channel synthesis filter bank. Two allow us to relate the vectors of the analysis and synthesis filter banks in a simple manner, the transfer functions $G_0(z)$ and $G_1(z)$ are expanded into a type 2 polyphase representation:

$$Go\ (z) = z^{-1}\ G_{oo}^{(p2)}(z^2) + G_{o1}^{(p2)}(z^2),$$

$$G_1\ (z) = z^{-1}\ G_{1o}^{(p2)}(z^2) + G_{11}^{(p2)}(z^2), \qquad (1.12)$$

$$\mathbf{G}^{(p2)}\ (z)]^{T} \cdot \begin{bmatrix} X_0(z) \\ X_1(z) \end{bmatrix} \qquad (1.13)$$

Where

$$\mathbf{G}^{(p2)}(z) = \begin{bmatrix} G_{oo}^{(p)}(z) & G_{o1}^{(p)}(z) \\ G_{10}^{(p)}(z) & G_{11}^{(p)}(z) \end{bmatrix} \qquad (1.14)$$

The matrix consists of polyphase type-2 component and is called polyphase matrix of synthesis filter.

Conditions for Perfect Reconstruction

If we connect the analysis and synthesis filter bank in series, we obtain the SBC filter bank as shown in fig below

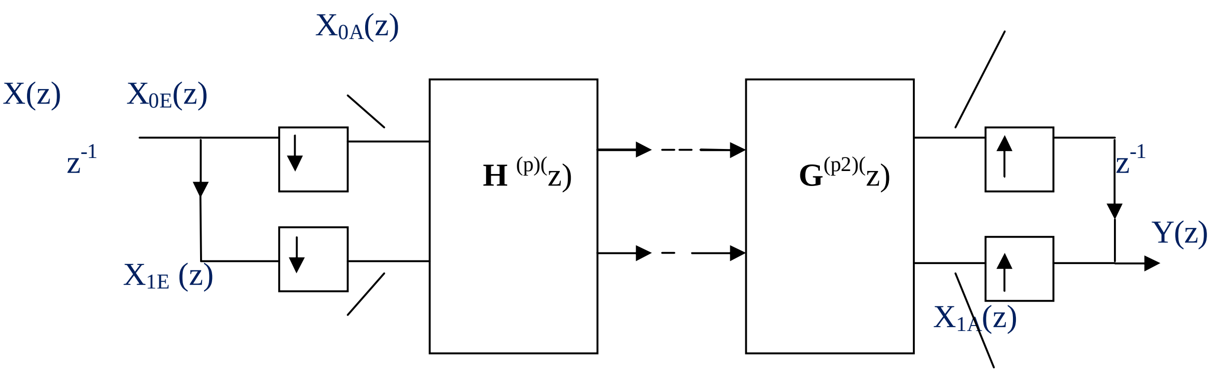

Figure 1.6.Two channel polyphase filter bank

The transfer characteristics of the series connection of both system is calculated from the above equation to give:

$$\begin{bmatrix} X_{0A}(z) \\ X_{1A}(z) \end{bmatrix} = [\mathbf{G}^{(p2)}\ (z)]^{T}.\ \mathbf{H}^{(p)}(z). \begin{bmatrix} X_{0E}(z) \\ X_{1E}(z) \end{bmatrix} \qquad (1.15)$$

Now the condition for perfect reconstruction is:

$\mathbf{G}^{(p2)}(z)]^{T}.\,\mathbf{H}^{(p)(}z) = z^{-k}I_2$ (1.16)

Where I_2 is the two- by two identity matrix. So in the matrix form we can write the above equation as

$$\begin{bmatrix} G_{oo}^{(p)}(z) & G_{o1}^{(p)}(z) \\ G_{10}^{(p)}(z) & G_{11}^{(p)}(z) \end{bmatrix} \begin{bmatrix} H_{oo}^{(p)}(z) & H_{o1}^{(p)}(z) \\ H_{10}^{(p)}(z) & H_{11}^{(p)}(z) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow 2z^{-M}H_{00}(z^{-1})H_{00}(z) = 1$$

Conclusion Drawn

Summarizing here is that the FIR filter can be efficiently represented by its polyphase components and use the polyphase component with aliasing free condition makes the system more efficient. As the polyphase structures for two fold decimation filter which is symmetric impulse response with order N requires about N/4 MPUs whether N is odd or even.

Designing of Two Channel QMF Using FIR Polyphase Component and M-Channel QMF

The design of optimal FIR analysis and synthesis QMF filters using polyphase structure is the main focus of this chapter, with the distortion measure to be minimized being a weighted mean square difference between the input and the reconstructed output. Unlike other design techniques, this algorithm assumes relationship between the all filters, therefore we have to choose only one filter coefficient. The input signal to the QMF bank is a discrete time signal, and the channel is assumed to be noiseless. All filters are FIR with even number of taps.

Analysis of the QMF bank with one polyphase component

We shall consider the system depicted in fig 4.1. The filters $H_O(z)$ and $H_1(z)$ are low pass and high pass FIR filter respectively with N, even number of taps. The time domain relationship between $h_O(n)$ and $e_o(n)$ & $e_1(n)$ imply the followings:

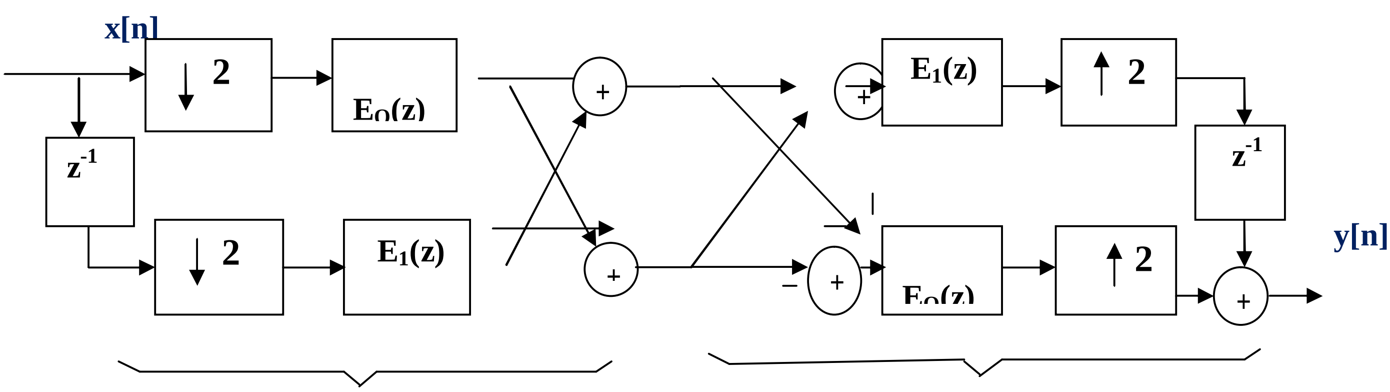

Figure 2.1.Quadrature mirror filter

$e_0(n) = h_0(2n)$ & $e_1(n) = h_0(2n+1)$ (2.1)

So both $e_0(n)$ and $e_1(n)$ are low pass filter with $e_o(n)$ is having the even coefficients and $e_1(n)$ is having the odd coefficients of $h_O(n)$. $H_0(z)$ can either be a Type 1 or a Type 2 linear phase FIR transfer function since it has to be a low pass filter. Then $h_0(n)$ satisfy the condition:

$$h_0 [n] = h_0 [N – n] \quad (2.2)$$

$H_0(e^{j\omega}) = e^{j\omega N/2} H_0(\omega)$

In the alias free QMF the distortion transfer function for the realization is given by:

$$T(z) = 2z^{-1}E_0(z^2)E_1(z^2) = 2z^{-1} \quad (2.3)$$

The resulting bank $H_0(z)$ is a linear phase FIR filter, then its polyphase components $E_0(z)$ and $E_1(z)$ are also linear phase FIR transfer functions. The frequency response of distortion transfer function can now written as

$$T(e^{j\omega}) = e^{-jN\omega} /2\{ | H_0(e^{j\omega}) |^2 – (-1)^N | H_0(e^{j(\Pi-\omega)}) |^2\} \quad (2.4)$$

From the above equation, it can be seen that if N is even, then $T(e^{j\omega}) = 0$ at the $\omega = \Pi/2$, implying severe amplitude distortion at the output of filter bank. N must be odd, in which case we have

$$T(e^{j\omega}) = e^{-jN\omega} /2\{ | H_0(e^{j\omega}) |^2 + | H_0(e^{j(\Pi-\omega)}) |^2\}= e^{-jN\omega} /2\{ | H_0(e^{j\omega}) |^2 + | H_1(e^{j\omega}) |^2\} \quad (2.5)$$

It follows from the above that the FIR 2 channel QMF bank will be of perfect reconstruction type if

$$| H_0(e^{j\omega}) |^2 + | H_1(e^{j\omega}) |^2 = 1 \quad (2.6)$$

Now, the 2 channel QMF bank with linear phase filter has no phase distortion, but will always exhibit amplitude distortion unless $| T(e^{j\omega}) |$ is a constant for all ω. If H0(z) is a very good low pass filter with $| H_0(e^{j\omega}) | = 1$ in the pass band and $| H_0(e^{j\omega}) | = 0$ in the stop band, then $H_1(z)$ is a very good high pass filter with its pass band coinciding with the stop band of $H_0(z)$ and vice-versa. As a result, $| T(e^{j\omega}) | = ½$ in the pass bands of $H_0(z)$ and$H_1(z)$ amplitude distortion occurs primarily in the transition band of these filters. Degree of distortion is determined by the amount of overlap between their squared-magnitude responses. This distortion can be minimized by controlling the overlap, which in turn can be controlled by appropriately choosing the pass band edge of $H_0(z)$.

The analysis filters $H_0(z)$ and $H_1(z)$ have typically a low-pass and high-pass frequency responses respectively, with a cutoff at $\Pi/2$

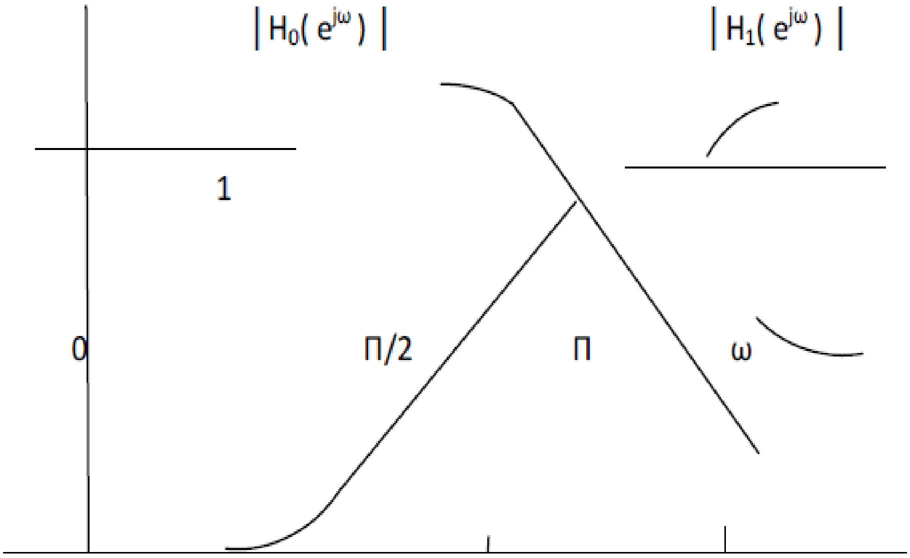

Figure 2.2 Frequency responses of LPF and HPF

One way to minimize the amplitude distortion is to iteratively adjust the filter coefficients $h_0(n)$of $H_0(z)$ on a computer such that

$|H_0(e^{j\omega})|^2 + |H_1(e^{j\omega})|^2 =$

(1) Stop band energy ω, and

(2) Square of error of $H_0(z)$ and $H_1(z)$

The objective function is given by:

$\Phi = \alpha\Phi 1 + (1-\alpha)\,\Phi_2$ (2.8)

Where

$\Phi_1 = \int_{\omega 1}^{\Pi} |H(e^{j\omega})|^2 d\,\omega$ (2.9)

and

$\Phi_2 = \int_{0}^{\Pi} (1 - |H_0(e^{j\omega})|^2 - |H_1(e^{j\omega})|^2)^2\, d\,\omega$ (2.10)

And

$0 < \alpha < 1$, and $\omega 1 = \Pi/2 + \xi$ for some small $\xi > 0$.

Different Design Algorithms

Hooke and Jeaves Algorithm for QMF design

The Hooke and Jeaves Algorithm is a search algorithm that attempts to minimize a single objective function of several variables. It requires a subroutine that takes the search variables as inputs and provides the value of objective function as its output.

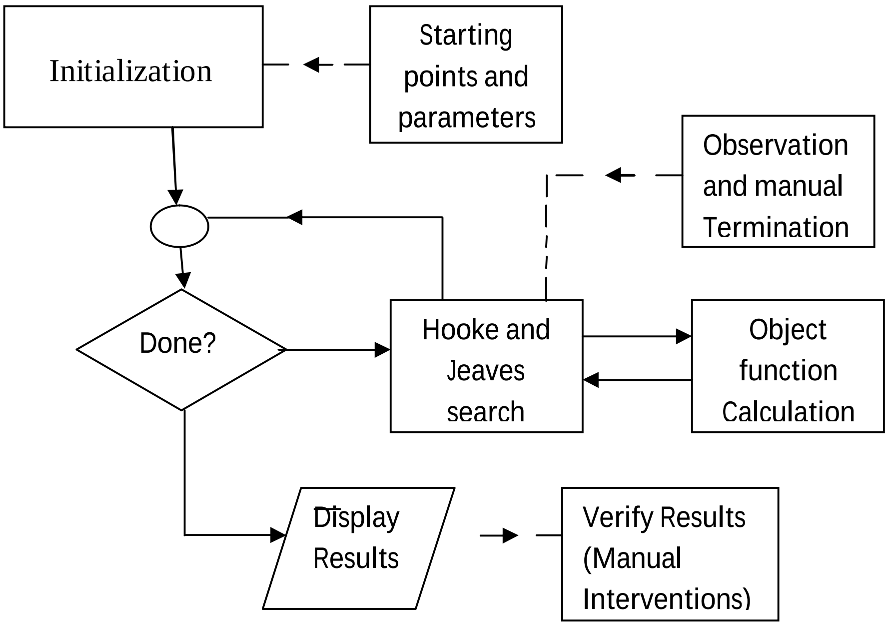

Figure 3.1.Optimization system block diagram

Due to the search method of Hooke and Jeaves, the starting point and starting increments are critical. A starting point that is too good may not minimize well because it will trap in local minima. If the step size is set to avoid these minima, it is usually too large that the relatively unsophisticated search will not find any better minima, and terminate. A starting point well away from the desired solution, on the other hand allows the algorithm a reasonably good probability of finding a useful minima. Manual intervention in the form of different and carefully selected starting points and careful observation of the run time output and results is necessary for success, because of many local minima.

Design of QMF Bank using the method given by A. Kumar

Over the past few years, several methods have been proposed to minimize amplitude distortion so that perfect reconstruction can be achieved. For the perfect reconstruction, the prototype filter in QMF bank must satisfy these conditions

$$\left|H_0(e^{j\omega})\right|^2 + \left|H_0(e^{j(\omega-\Pi/2)})\right|^2 = 1 \text{ for } 0 < \omega < \Pi/2 \quad (3.1)$$

$$\left|H_0(e^{j\omega})\right| = 0 \quad \text{for } \omega > \Pi/2 \quad (3.2)$$

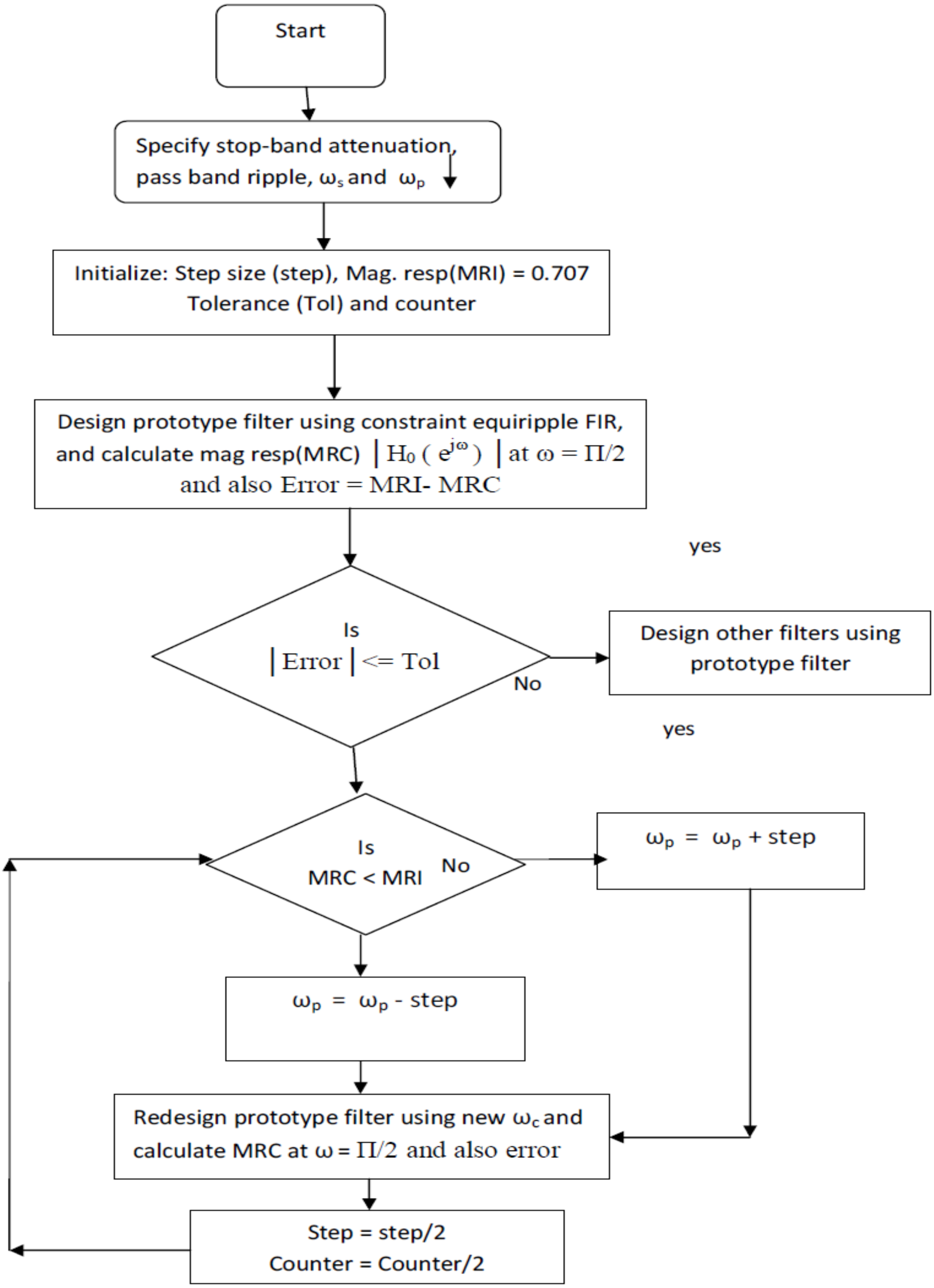

Figure 3.2.Design algorithm for optimization of QMF

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If $\Phi(e^{j\omega}) = |H_0(e^{j\omega})|^2 + |H_0(e^{j(\omega-\Pi)})|^2$, then for the perfect reconstruction, $Q(e^{j\omega}) = 1$ and if it is evaluated at $\omega = 0.5\,\Pi$ then this implied that the magnitude response of the prototype filter is 0.707 in ideal condition.

$|H_0(e^{j0.5\Pi})|^2 = 0.5$ or $|H_0(e^{j0.5\Pi})| = 0.707$ (3.3)

In QMF bank, the amplitude distortion also depends on the degree of overlapping between both the analysis filters. If the passband edge frequency is taken either too large or too small, then the reconstruction error is large at $\omega = 0.5\,\Pi$. Therefore optimum ω_p is to be selected such that the reconstruction error is minimum. Here in proposed algorithm, the optimum ω_p is found by adjusting filter coefficient by eqn. (3.3).

In this algorithm filter order (N) and stop band attenuation (A_s) are fixed, and instead of cut-off frequency, pass band edge frequency (ω_p) is optimized so that amplitude distortion is reduced. The prototype filter is designed using given specifications before the optimization start. The magnitude response (MRC) of the prototype filter is evaluated $|H_0(e^{j\omega})|$ at $\omega = 0.5\,\Pi$ and error between ideal magnitude response (MRI) and calculated MRC is to be calculated. If the tolerance (TOL) is not satisfied, then ω_p is varied using the step size. If MRC is less then MRI, then ω_p is varied using the step size otherwise it is decreased. The prototype filter is redesigned using new ω_p, same order, and same A_s. In every iteration, step size is halved. Therefore, this approach can be used for the design of prototype filter of larger taps. Here the prototype filter is designed using the constraint equiripple FIR technique.

Conclusion and Future Scope

Conclusion

There are several classes of optimal filter according to choice of performance measure or objective function to be minimized. FIR filter based on window approach do not yield the optimum filter .These filters suffers from Gibbs phenomenon. Also least square approach includes matrix inversions which are difficult and time consuming. So the approach of this thesis work is more efficient from other approaches used for error optimization. The adjusting parameter α is introduced in this thesis to minimize the error. As α is increasing the peak stop band ripple is reduced at the expense of peak pass band ripple. So we conclude that by adjusting the value of α we can adjust the error received.

Future Scope

The optimization algorithm is an iterative procedure which alternates between computing a pair of optimal analysis filters and optimal synthesis filters, until the error reduction is negligible. The extension of this algorithm for designing of M band QMF is the most obvious area for future research. The extension should be quite easy, since the optimization problem is a least square one. In M band case, care must be taken as to how many of the filters can be fixed. It would also be useful to improve the frequency band separation of the filters, in the case that one of the filter is fixed.

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